

問1:

(1) 右の線形計画問題を  
不等式標準形に  
書き直せ.

$$\begin{aligned} \text{最大化: } & 2x + 2y + 3z \\ \text{条件: } & 5x + 3z \leq 8 \\ & 2z = 2 \\ & 4y + z \geq 9 \\ & x, y \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最大化: } & 2x + 2y + 3(z' - z'') \\ \text{条件: } & 5x + 3(z' - z'') \leq 8 \\ & 2(z' - z'') = 2 \\ & 4y + (z' - z'') \geq 9 \\ & x, y, z', z'' \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -2x - 2y - 3(z' - z'') \\ \text{条件: } & 5x + 3(z' - z'') \leq 8 \\ & 2(z' - z'') \geq 2 \\ & 2(z' - z'') \leq 2 \\ & 4y + (z' - z'') \geq 9 \\ & x, y, z', z'' \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -2x - 2y - 3(z' - z'') \\ \text{条件: } & 5x + 3(z' - z'') \leq 8 \\ & 2(z' - z'') = 2 \\ & 4y + (z' - z'') \geq 9 \\ & x, y, z', z'' \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -2x - 2y - 3(z' - z'') \\ \text{条件: } & -5x - 3(z' - z'') \geq -8 \\ & 2(z' - z'') \geq 2 \\ & -2(z' - z'') \geq -2 \\ & 4y + (z' - z'') \geq 9 \\ & x, y, z', z'' \geq 0 \end{aligned}$$

(2) 右の線形計画問題を  
不等式標準形および  
等式標準形に  
書き直せ.

$$\begin{aligned} \text{最大化: } & 3x + 6y \\ \text{条件: } & x + y = 2 \\ & x + 4y \leq 2 \\ & y \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最大化: } & 3(x' - x'') + 6y \\ \text{条件: } & (x' - x'') + y = 2 \\ & (x' - x'') + 4y \leq 2 \\ & x', x'', y \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & (x' - x'') + y \leq 2 \\ & (x' - x'') + y \geq 2 \\ & (x' - x'') + 4y \leq 2 \\ & x', x'', y \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & (x' - x'') + y = 2 \\ & (x' - x'') + 4y \leq 2 \\ & x', x'', y \geq 0 \end{aligned}$$

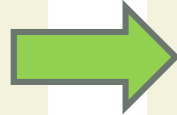
$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & -(x' - x'') - y \geq -2 \\ & (x' - x'') + y \geq 2 \\ & -(x' - x'') - 4y \geq -2 \\ & x', x'', y \geq 0 \end{aligned}$$

不等式標準形

(2) 右の線形計画問題を不等式標準形および  
等式標準形に書き直せ.

### 不等式標準形から等式標準形へ

$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & -(x' - x'') - y \geq -2 \\ & (x' - x'') + y \geq 2 \\ & -(x' - x'') - 4y \geq -2 \\ & x', x'', y \geq 0 \end{aligned}$$



$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & -(x' - x'') - y - a = -2 \\ & (x' - x'') + y - b = 2 \\ & -(x' - x'') - 4y - c = -2 \\ & x', x'', y \geq 0, a, b, c \geq 0 \end{aligned}$$

### 不等式標準形導出の途中の形から等式標準形へ

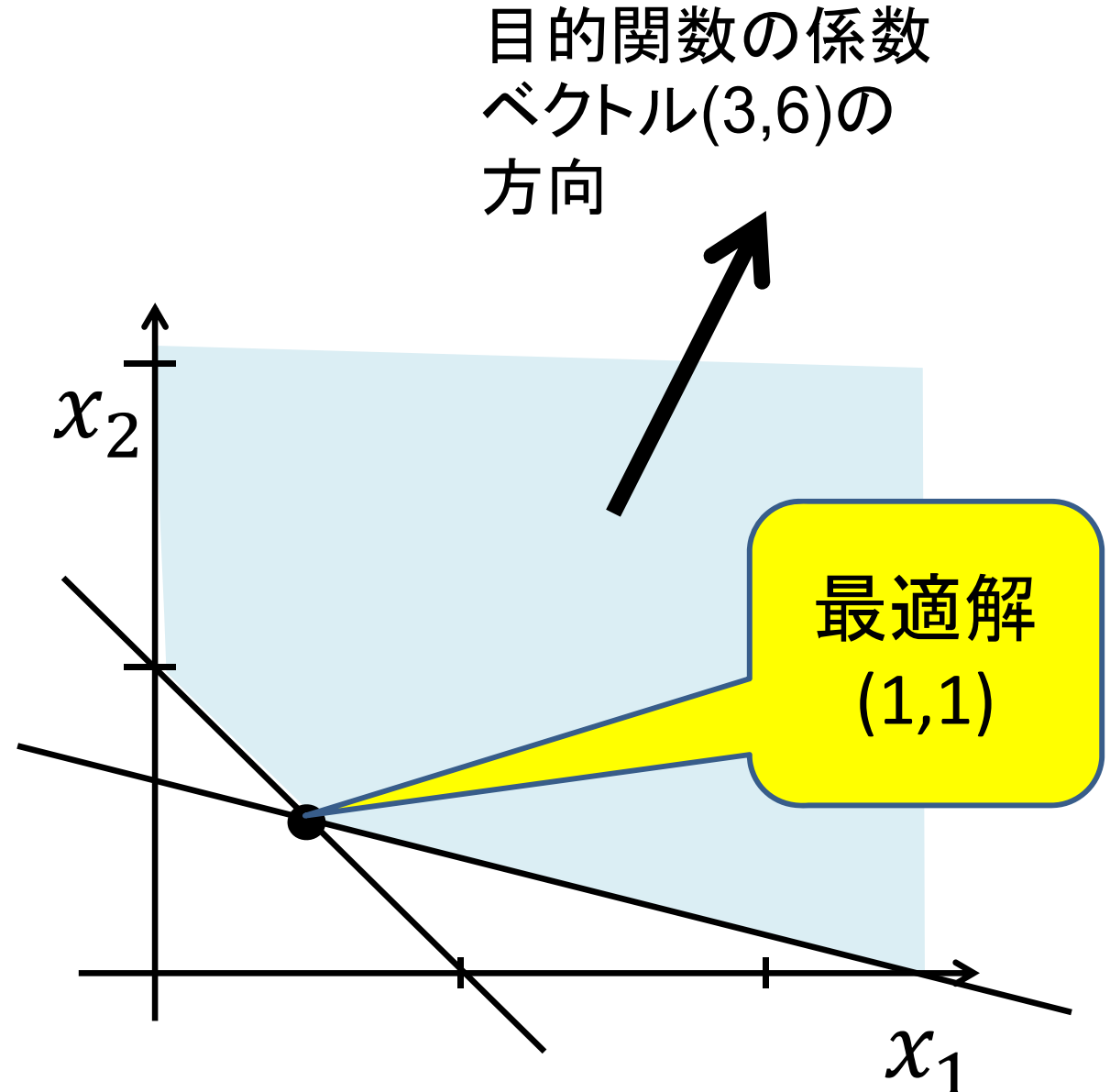
$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & (x' - x'') + y = 2 \\ & (x' - x'') + 4y \leq 2 \\ & x', x'', y \geq 0 \end{aligned}$$



$$\begin{aligned} \text{最小化: } & -3(x' - x'') - 6y \\ \text{条件: } & (x' - x'') + y = 2 \\ & (x' - x'') + 4y + a = 2 \\ & x', x'', y \geq 0, a \geq 0 \end{aligned}$$

問2:(1) 下記の線形計画問題の実行可能領域を図示し、最適解を求めなさい。

最小化  $3x_1 + 6x_2$   
条件  $x_1 + x_2 \geq 2$   
 $x_1 + 4x_2 \geq 5$   
 $x_1 \geq 0, x_2 \geq 0$



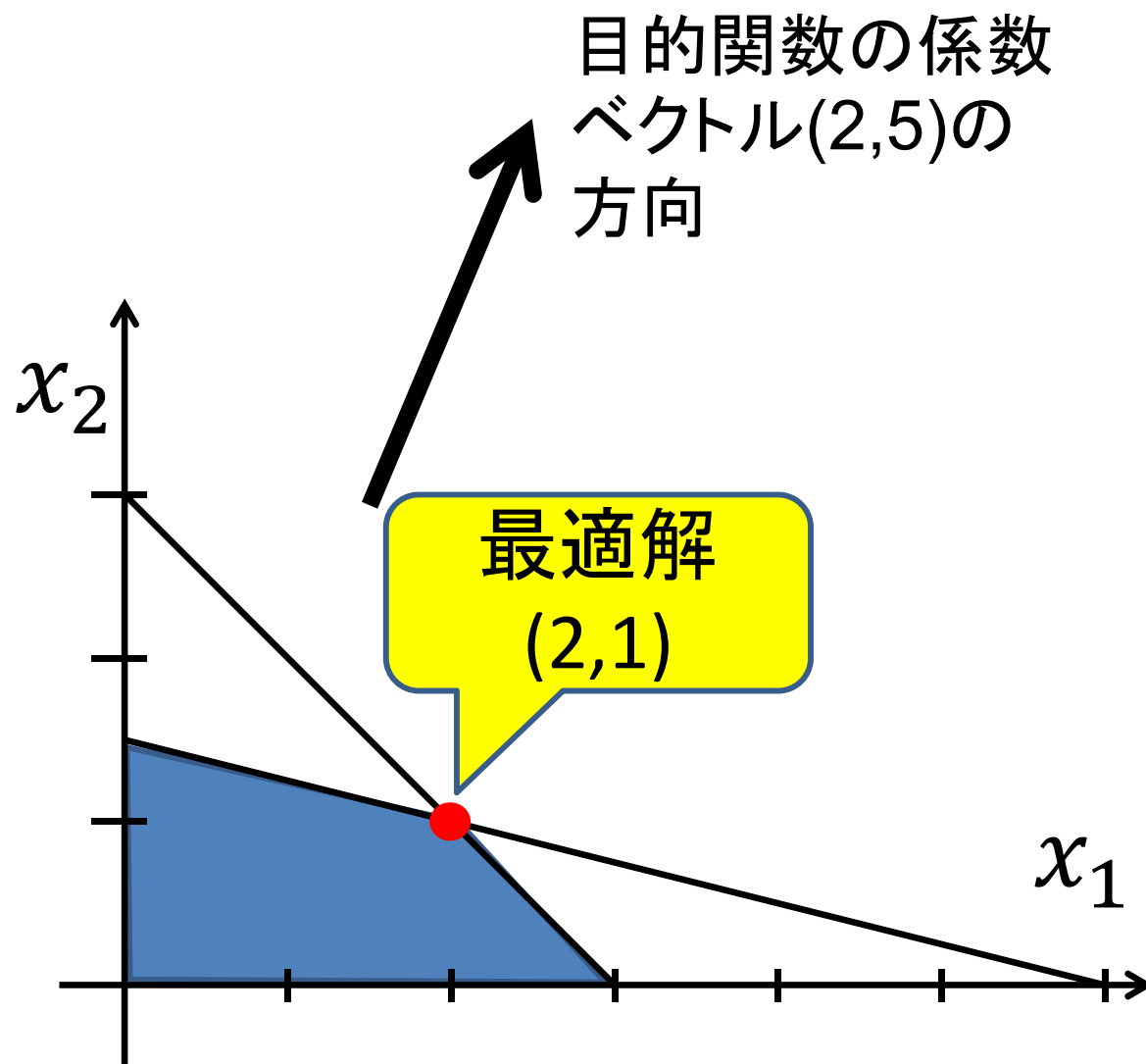
## 問2:

(2) 上記の線形計画問題の双対問題を求めなさい.

(3) 双対問題の実行可能領域を図示し, 最適解を求めなさい.

$$\begin{array}{ll} \text{最小化} & 3x_1 + 6x_2 \\ \text{条件} & x_1 + x_2 \geq 2 \\ & x_1 + 4x_2 \geq 5 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{最大化:} & 2y_1 + 5y_2 \\ \text{条件:} & y_1 + y_2 \leq 3 \\ & y_1 + 4y_2 \leq 6 \\ & y_1, y_2 \geq 0 \end{array}$$



問3: 双対問題の双対問題は主問題に一致する事を証明せよ.

$$\begin{aligned} \text{最大化: } & b_1y_1 + b_2y_2 + \cdots + b_my_m \\ \text{条件: } & a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m \leq c_1 \\ & a_{12}y_1 + a_{22}y_2 + \cdots + a_{m2}y_m \leq c_2 \\ & \vdots \\ & a_{1n}y_1 + a_{2n}y_2 + \cdots + a_{mn}y_m \leq c_n \\ & y_1, y_2, \dots, y_m \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & -b_1y_1 - \cdots - b_my_m \\ \text{条件: } & -a_{11}y_1 - \cdots - a_{m1}y_m \geq -c_1 \\ & \vdots \\ & -a_{1n}y_1 - \cdots - a_{mn}y_m \geq -c_n \\ & y_1, y_2, \dots, y_m \geq 0 \end{aligned}$$

不等式標準形  
へ変換

双対問題をつくる

$$\begin{aligned} \text{最大化: } & -c_1z_1 - \cdots - c_nz_n \\ \text{条件: } & -a_{11}z_1 - \cdots - a_{1n}z_n \leq -b_1 \\ & \vdots \\ & -a_{m1}z_1 - \cdots - a_{mn}z_n \leq -b_m \\ & z_1 \geq 0, \dots, z_n \geq 0 \end{aligned}$$

問3: 双対問題の双対問題は主問題に一致する事を証明せよ.

$$\begin{aligned} \text{最大化: } & -c_1z_1 - \cdots - c_nz_n \\ \text{条件: } & -a_{11}z_1 - \cdots - a_{1n}z_n \leq -b_1 \\ & \vdots \\ & -a_{m1}z_1 - \cdots - a_{mn}z_n \leq -b_m \\ & z_1 \geq 0, \dots, z_n \geq 0 \end{aligned}$$

$$\begin{aligned} \text{最小化: } & c_1z_1 + \cdots + c_nz_n \\ \text{条件: } & a_{11}z_1 + \cdots + a_{1n}z_n \geq b_1 \\ & \vdots \\ & a_{m1}z_1 + \cdots + a_{mn}z_n \geq b_m \\ & z_1 \geq 0, \dots, z_n \geq 0 \end{aligned}$$

不等式標準形  
へ変換

変数  $z_j$  を  $x_j$  に置き換えると,  
主問題と全く同じになる