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Female Labor Participation, Fertility, and Structural Change

Takeo Hori



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Tokyo Institute of Technology

2-12-1 Ookayama, Meguro-ku, Tokyo 152-8552, JAPAN
<http://educ.titech.ac.jp/iee/>

Female Labor Participation, Fertility, and Structural Change *

Takeo Hori[†]

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Abstract

The rise of service economy may affect female labor participation as well as fertility. We construct a model where market services can be used for childrearing. We show that an increase in availability of market childrearing services increases fertility as well as female labor participation. The rise of service economy may explain differences in fertility rate among developed countries.

Keywords: service economy, fertility dynamics, childrearing, technological progress.

JEL Classification Numbers: J1, J16, L16.

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[†]Department of Industrial Engineering and Economics, School of Engineering, Tokyo Institute of Technology, 2-12-1, Ookayama, Meguro-ku, Tokyo 152-8552, JAPAN. E-mail: hori.t.ag@m.titech.ac.jp

1 Introduction

In developed countries, female labor participation rates have been increasing since the 1970s (see Figure 1). For example, female labor participation rate increased from 49% to 68% between 1970 and 2010 in the United States. During the same period, developed countries experienced decline in fertility rates, as show in Figure 2. Total fertility rate in the United States decreased from 2.48 to 1.93.

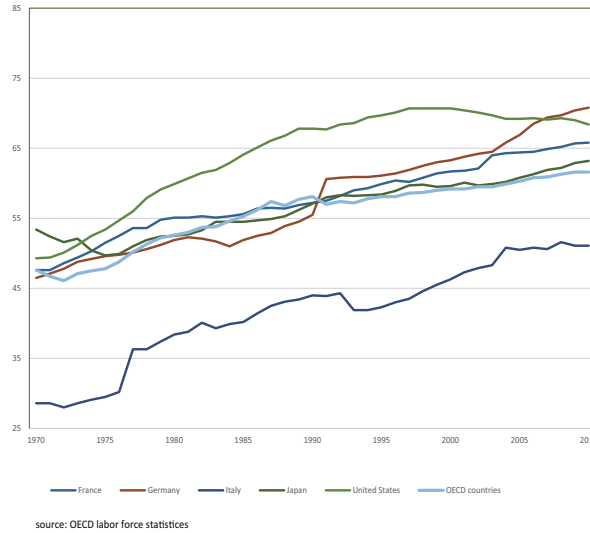


Figure 1: Female Labor Participation Rate

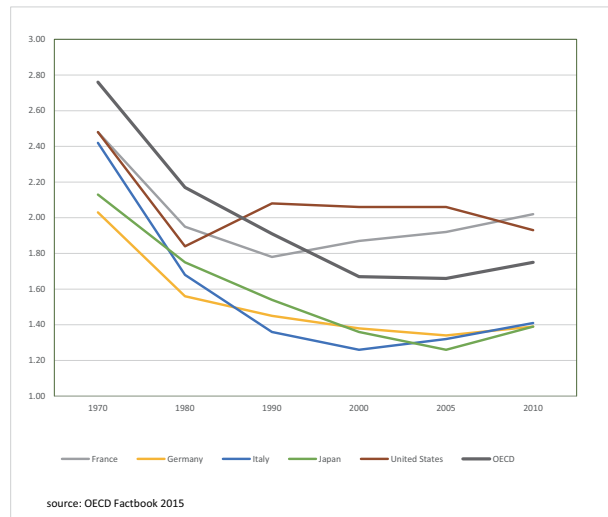


Figure 2: Total Fertility Rate

These trends in female labor participation and fertility are observed commonly in developed countries. However, Figure 2 shows that different countries follow different fertility dynamics. It seems that we can divide developed countries into at least two groups according to their fertility dynamics. The first group includes countries like France and the United States. In these countries, fertility decline seems to have stopped around 1990. After that, fertility has remained fairly constant or has even increased. In 2010, total fertility rates of countries in this group are nearly 2. The

second group includes countries like Germany and Japan, where total fertility rates are as low as 1.4 in 2010.

The present study argues that differences in fertility dynamics or total fertility rate among developed countries might be associated with the growth of service economy. Figure 3 plots total fertility rate against share of services. This figure shows a clear positive relationship between share of services and fertility; fertility rate is high in countries with high shares of services. Even if we use market service shares, the similar positive relationship is obtained in Figure 4. This relationship may be interpreted as follow. As the rise of service economy, the market childbearing services would have become more available, which could have been reducing the burden of childrearing of female. Hence, fertility have remained fairly high at the same time as female labor participation has risen.

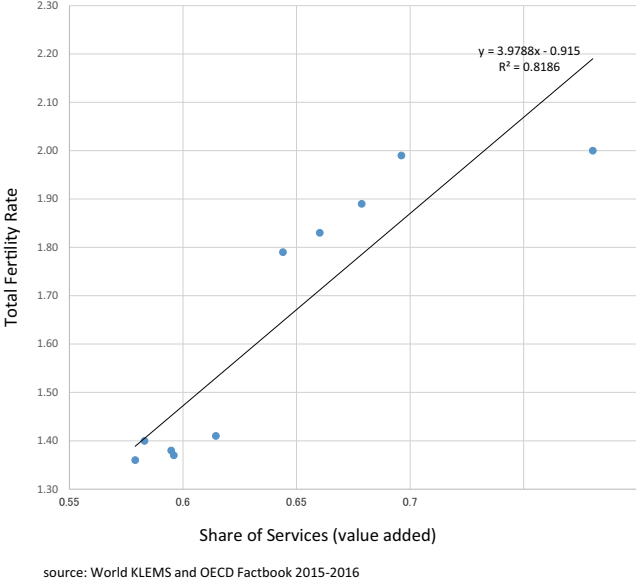


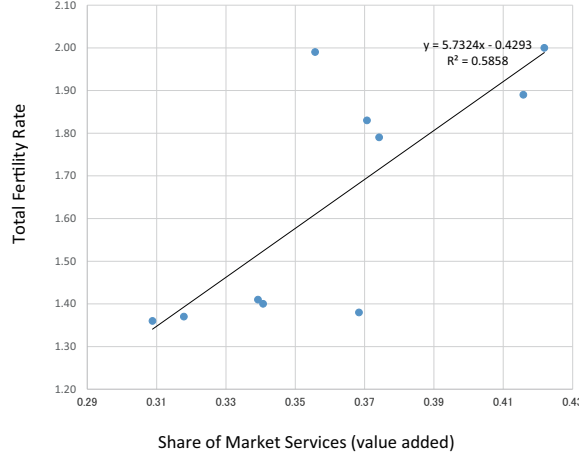
Figure 3: Services Share (value-added) vs. Total Fertility Rate (2009)
Austria, Belgium, Spain, France, Germany, Italy, Japan, Netherlands, United Kingdom, United States

To confirm this intuition, we develop a theoretical model in which there are two production sectors, services and goods. Childrearing requires female’s and male’s time. Besides, market services can be used for childrearing. We show that an increase in availability of market childrearing services increases fertility as well as female labor participation.

The present study is related to Ngai and Petrongolo (2014) who show that the rise of service economy may account for some part of increases in female labor participation in the United States. In contrast to Ngai and Petrongolo (2014), we consider the effects of service economy on fertility.

Organization of the Paper

Section 2 presents our model . We also present the partial equilibrium results because they are useful to understand the mechanism of the model. Section 3 considers the general equilibrium and shows that the availability of market childrearing services affects the dynamics of female market labor and fertility. Our concluding remarks are presented in Section 4.



source: World KLEMS and OECD Factbook 2015-2016

Figure 4: Market Services Share (value-added) vs. Total Fertility Rate (2009)
Austria, Belgium, Spain, Finland, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom, United States

2 The Model and Partial Equilibrium

2.1 Households

The economy is populated by one unit of representative household. The utility function of the representative household is

$$U = \ln D + \xi \ln n, \quad 0 \leq \xi < 1, \quad (1)$$

where D is a composite of goods and services consumption:

$$D = \left[\zeta_C C^{\frac{\eta-1}{\eta}} + \zeta_D S_D^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad 0 < \eta < 1, \quad 0 < \zeta_C, \zeta_D < 1, \quad \text{and} \quad \zeta_C + \zeta_D = 1. \quad (2)$$

C denotes consumption of goods while S_D denotes consumption of services. We assume that goods and services are poor substitute in (1), $0 < \eta < 1$. The number of children that the representative household has is denoted as n and satisfies

$$n = \left[\theta_h h^{\frac{\gamma-1}{\gamma}} + \theta_S S_n^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad \gamma > 1, \quad 0 < \theta_C, \theta_S < 1, \quad \text{and} \quad \theta_h + \theta_S = 1, \quad (3)$$

where S_n is the purchase of market services for childrearing and h is the home production of childrearing services. We assume that $\theta_S \in (0, 1)$, except for Subsection 3.2 that considers a special case where $\theta_S = 0$. We interpret market childrearing services broadly so that they include any services that reduce the burden of childrearing of households. The assumption $\gamma > 1$ means that childrearing at home can be substituted by market childrearing services. A large θ_S means that (i) there are sufficient market childrearing services available to households and (ii) these services are highly productive in childrearing.

Childrearing at home is done by using male and female labor, l_m and l_f :

$$h = \left[\omega_m l_m^{\frac{\epsilon-1}{\epsilon}} + \omega_f l_f^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0, \quad \omega_m + \omega_f = 1, \quad \text{and} \quad \frac{1}{2} < \omega_f \leq 1. \quad (4)$$

The assumption, $1/2 < \omega_f \leq 1$, means that the domestic childrearing is intensive in female labor. The budget constraint of the household is give by

$$p_C C + p_S (S_D + S_n) = w_m (L_m - l_m) + w_f (L_f - l_f), \quad (5)$$

where p_C (p_S) is the price of consumption goods (services), w_m (w_f) is the wage rate for male (female), and L_m (L_f) is the time endowment of male (female). We denote the total expenditure on consumption of goods and market services as E_D :

$$p_C C + p_S S_D = E_D (= P_D D), \quad (6)$$

where P_D is the price index for D (see (10)). We rewrite (5) as

$$P_D D + p_S S_n = w_m (L_m - l_m) + w_m (L_f - l_f). \quad (7)$$

Optimization

We solve the utility maximization problem of households in two steps. The first step maximizes (2) subject to (6), which yields

$$C = \frac{P_D D \zeta_C^\eta p_C^{-\eta}}{\zeta_C^\eta p_C^{1-\eta} + \zeta_S^\eta p_S^{1-\eta}}, \quad (8)$$

$$S_D = \frac{P_D D \zeta_S^\eta p_S^{-\eta}}{\zeta_C^\eta p_C^{1-\eta} + \zeta_S^\eta p_S^{1-\eta}}, \quad (9)$$

$$P_D = \left[\zeta_C^\eta p_C^{1-\eta} + \zeta_S^\eta p_S^{1-\eta} \right]^{\frac{1}{1-\eta}}. \quad (10)$$

The following discussion normalizes the price of D to one, $P_D = 1$.

The second step maximizes (1) subject to (3), (4), and (7). Appendix A shows that the second step yields

$$P_D D = \frac{1}{1 + \xi} (w_m L_m + w_f L_f), \quad (11)$$

$$\frac{h}{S_n} = \left(\frac{\theta_h}{\theta_S} p_S \right)^\gamma \left[\frac{\omega_m^\epsilon}{w_m^{\epsilon-1}} + \frac{\omega_f^\epsilon}{w_f^{\epsilon-1}} \right]^{\frac{\gamma}{\epsilon-1}} \equiv H(p_S, w_m, w_f), \quad (12)$$

$$S_n = \frac{\xi}{(1 + \xi) p_S} \frac{w_m L_m + w_f L_f}{1 + \frac{\theta_h}{\theta_S} H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}} \equiv S_n(p_S, w_m, w_f), \quad (13)$$

$$h = H(p_S, w_m, w_f) S_n(p_S, w_m, w_f). \quad (14)$$

The price of services, p_S , and the wage rates, w_m and w_f , affect the demand for domestic and market childrearing services. If we solve the following two equations, l_f and l_m are determined:

$$l_m = \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon l_f. \quad (15)$$

$$w_m l_m + w_f l_f = \frac{\xi}{1 + \xi} (w_m L_m + w_f L_f) \frac{\frac{\theta_h}{\theta_S} H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}}{1 + \frac{\theta_h}{\theta_S} H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}}, \quad (16)$$

Thus, p_S , w_m and w_f also affect the time allocated to domestic childrearing, l_m and l_f . The number of children that the household has is also a function of p_S , w_m , and w_f :

$$n = \frac{\xi}{(1 + \xi) p_S} (w_m L_m + w_f L_f) \left[\theta_S + \theta_h H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma-1}}. \quad (17)$$

Implications

Before we proceed to the production side of the economy, we present partial equilibrium implications of the households' behavior. We first focus on the demand for services relative to goods.

Result 1 Suppose that goods and services are poor substitute in (1), $0 < \eta < 1$. Then, an increase in relative price of services, p_S/p_C , increases the expenditure on service consumption relative to that on goods consumption.

(Proof) From (8) and (9), we obtain

$$\frac{p_S S_D}{p_C C} = \left(\frac{\zeta_S}{\zeta_C} \right)^\eta \left(\frac{p_S}{p_C} \right)^{1-\eta}. \quad (18)$$

The assumption $0 < \eta < 1$ ensures Result 1. \square

Result 1 is fairly standard and shows that a change in relative price triggers a demand shift from goods to services.

We next turn to households' behavior concerning fertility decisions.

Result 2 (i) Increases in the wage rates, w_m and w_f , and decreases in service price, p_S , have a negative effect on the relative demand for domestic childrearing services, h/S_n . (ii) Suppose that the elasticity of substitution between h and S_n in n is sufficiently high ($\gamma > 1$). Increases in w_m and w_f and decreases in p_S have positive effects on demand for market childrearing services, S_n , and expenditure on market childrearing services, $p_S S_n$.

(Proof) Differentiating (12) and (13) yields Result 2.

Intuition behind Result 2 is as follows. An increase in the wage rates, w_m and w_f , makes domestic childrearing expensive. Since the elasticity of substitution between h and S_n is high, households increase their dependence of childrearing on market services.

The next result shows that how domestic childrearing time of female responds to prices.

Result 3 Suppose that the elasticity of substitution between h and S_n in n is sufficiently high ($\gamma > 1$) and female does more childrearing than male ($l_f/L_f > l_m/L_m$). Then, an increase in w_f reduces l_f while an increase in p_S increases l_f . An increase in w_m has an ambiguous effect on l_f .

(Proof) See Appendix B.

Result 3 is intuitive. An increase in w_f raises the costs of domestic childrearing of female. Thus, female childrearing time reduces. When the price of market services increases, the use of market childrearing services decreases, and then female childrearing time increases.

Before examining the effects on fertility n , we consider the female market labor participation.

Result 4 Suppose that all the conditions of Result 3 hold and that the following condition holds:

$$\frac{L_m - l_m}{L_f - l_f} > \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon. \quad (19)$$

Then, an increase in w_f has a positive effect on the relative market labor of female, $(L_f - l_f)/(L_m - l_m)$.

(Proof) See Appendix C.

The condition (19) tends to hold when (i) male does much market labor relative to female, (ii) the wage rate for female labor is low relative to that of male, and (iii) the productivity of female in domestic childrearing is high relative to that of male. These conditions are fairly reasonable. Therefore, under reasonable conditions, an increase in w_f stimulates female market labor participation.

Finally, Result 5 shows how fertility is affected by w_f .

Result 5 Suppose that the following inequality holds:

$$\frac{L_f}{L_m} < \left(\frac{\omega_f w_m}{\omega_m w_f} \right)^\epsilon. \quad (20)$$

Then, if $\theta_S (= 1 - \theta_h \in [0, 1])$ is sufficiently small, an increase in w_f reduces n . If θ_S is sufficiently large, an increase in w_f increases n .

(Proof) See Appendix D.

The condition (20) tends to hold when (i) the wage rate for female labor is low relative to that of male and (ii) the productivity of female in domestic childrearing is high relative to that of male. These conditions are fairly reasonable.

Result 5 implies that if market childrearing services are available and productive (θ_S is large), an increase in w_f raises fertility. The intuition is simple. As Result 2 show, an increase in w_f raises S_n . Because θ_S is large, n also increases. In contrast, if market childrearing services are not so available and productive (θ_S is small), an increase in w_f reduces fertility.

We now summarize the partial equilibrium results of the household behaviors.

Result 6 Suppose that Results 2-5 holds. When w_f increases, (i) female market labor increases relative to male market labor, (ii) the usage of market childrearing services increases, and (iii) at the same time,

1. fertility decreases if θ_S is small.
2. fertility increases if θ_S is large.

Whether fertility increases or decreases with female market labor depends on availability and productivity of market childrearing services.

2.2 Production

Production function of Sector $i (= C \text{ or } S)$ is

$$y_i = A_i \left[\mu_{m,i} L_{m,i}^{\frac{\sigma-1}{\sigma}} + \mu_{f,i} L_{f,i}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \quad 0 < \mu_{m,i}, \mu_{f,i} < 1, \quad \text{and} \quad \mu_{m,i} + \mu_{f,i} = 1 \quad (21)$$

where $i = C$ and $i = S$ indicate goods production sector and service sector, respectively, and A_i is the total factor productivity (TFP) of sector i . We have $y_C = C$ and $y_S = S (\equiv S_D + S_n)$. In production of both goods and services, male and female labor substitute each other ($\sigma > 1$). We assume that female labor has comparative advantage in services production:

$$\mu_{f,s} > \mu_{f,c}. \quad (22)$$

Profit Maximization

The profits maximization of each sector yields

$$p_i y_i^{\frac{1}{\sigma}} \mu_{k,i} L_{k,i}^{-\frac{1}{\sigma}} = w_k, \quad (23)$$

where $i = C$ or S and $k = m$ or f . From (23), we derive

$$\frac{L_{f,i}}{L_{m,i}} = \left(\frac{\mu_{f,i} w_m}{\mu_{m,i} w_f} \right)^\sigma, \quad (24)$$

$$L_{k,i} = \frac{p_i y_i \mu_{k,i}^\sigma w_k^{-\sigma}}{\mu_{m,i}^\sigma w_m^{1-\sigma} + \mu_{f,i}^\sigma w_f^{1-\sigma}}. \quad (25)$$

We substitute (25) into (21) to derive

$$p_i = \frac{1}{A_i} \left[\mu_{m,i}^\sigma w_m^{1-\sigma} + \mu_{f,i}^\sigma w_f^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (26)$$

Both the prices of goods and services depend on the wage rates, w_m and w_f .

Implication

As in the previous subsection, we explore the partial equilibrium implications of the production side of the economy.

Result 7 Suppose that female labor has comparative advantage in services production, $\mu_{f,s} > \mu_{f,c}$.

1. An increase in w_f/w_m has a positive effect on the relative price of services, p_s/p_c , if $\sigma \neq 1$.
2. The relative labor demand of services for both sexes, $L_{k,s}/L_{k,c}$ ($k = m$ or f), is positively affected by
 - (a) the relative services expenditure of households, $p_s S_t / p_c C$, where $S_t = S_D + S_n$,
 - (b) the relative wage of female labor, w_f/w_m , if male and female labor substitute each other ($\sigma > 1$).

(Proof) See Appendix E.

The intuition of Result 7-1 is as follows. Service sector is more intensive in female labor than goods sector ($\mu_{f,s} > \mu_{f,c}$). Thus, an increase in w_f/w_m positively affects the relative price of services, p_s/p_c . The intuition of Result 7-2-(a) holds because an increase in $p_s S_t / p_c C$ stimulates the production of service sector. Finally, an increase in w_f/w_m raises p_s/p_c , which also stimulates production in services.

Summary of the Partial Equilibrium Results

Result 6 suggests that the labor wage rates affect female labor participation and fertility decision. Result 7, together with Result 1, suggests that structural change among production sectors is relevant to the labor wage rates. Thus, the results so far suggest that female labor participation and fertility are closely associated with structural change. However, we obtain these results by analyzing household sector and production sector separately and so far we ignore the factors that determine the labor wage rates. To study interaction between structural change and the dynamics of female labor and fertility, the following discussion considers a general equilibrium framework and then observes that availability and productivity of market childrearing services have important roles.

3 General Equilibrium

3.1 Labor Market

Labor market clears as

$$L_m = L_{m,c} + L_{m,s} + l_m, \quad (27)$$

$$L_f = L_{f,c} + L_{f,s} + l_f. \quad (28)$$

Appendix F shows that $L_{k,c}$, $L_{k,s}$, and l_k ($k = m$ or f) are all functions of w_f/w_m . Thus, (28) determines equilibrium value of w_f/w_m , while one of (27) and (28) is redundant. Once w_f/w_m is

obtained, (E.1) determines p_S/p_C . Then, $P_D = 1$ and (10), together with p_S/p_C , determine the values of p_S and p_C . If we use the values of w_f/w_m , p_S , and p_C in (26), we obtain w_f and w_m . Since all prices, w_f , w_m , p_S , and p_C are derived, we obtain a general equilibrium.

3.2 A Special Case: $\theta_S = 0$

This subsection focuses on a case where no market childrearing services are available, which means that $\theta_S = 0$. This special case allows for an analytical solution. Hence, we easily interpret the results and the mechanism behind them. As a driving force of structural change, we consider biased technical change such that the relative productivity of goods producing sector, A_C/A_S , increases, following Ngai and Pissarides (2007). We obtain the following general equilibrium result.

Proposition 1 *Suppose that $L_m = L_f$, $\omega_m < \omega_f$, $\sigma > 1 > \eta > 0$, and $\mu_{f,S} > \mu_{f,C}$ and that $\theta_S = 0$. Then, there exists at least one general equilibrium.*

Suppose further that the equilibrium is unique. Then, when A_C/A_S increases,

1. *the relative wage for female, w_f/w_m , increases;*
2. *the relative price of services, p_S/p_C , increases;*
3. *the relative expenditure on services, $p_S S/p_C C$, increases;*
4. *the relative labor demand in service sector, $L_{k,S}/L_{k,C}$, increases for both sexes ($k = m$ or f);*

Moreover, suppose that the wage rate for female is lower than that for male, $w_f/w_m < 1$, at equilibrium. When A_C/A_S increases,

5. *the domestic childrearing of female, l_f , decreases;*
6. *the relative market labor of female, $(L_f - l_f)/(L_m - l_m)$, increases if (19) holds;*
7. *fertility, n , decreases if (20) holds.*

(Proof) See Appendix G.

Proposition 1 does not show the uniqueness of equilibrium. However, we confirm numerically that under a wide range of parameter values, equilibrium is unique.

The mechanism behind Proposition 1 is as follows. Remember that goods and services are poor substitutes ($\eta < 1$) in the utility of households. Thus, if service sector becomes less productive, production factors are reallocated away from more productive goods sector to less productive service sector, which is represented by an increase in $L_{k,S}/L_{k,C}$. Since female has comparative advantage in service production ($\mu_{f,S} > \mu_{f,C}$), the shift in production factors toward services stimulates the labor demand for female relatively to that for male, which rises the relative wage for female w_f/w_m . Since service sector is more intensive in female labor than goods sector ($\mu_{f,S} > \mu_{f,C}$), an increase in w_f/w_m has a positive effect on the relative price of services p_S/p_C . Besides, a relative decline in productivity of service sector also has an increasing effect on p_S/p_C . Thus, service price increases relatively to goods price. Accordingly, households increase expenditure on services relatively to that on goods because goods and services are poor substitutes ($\eta < 1$).

An increase in the relative wage for female w_f/w_m also affects labor supply of households. With a raise in the relative wage for female, the market labor of female increases and female domestic childrearing decreases. Since no market childrearing services are available ($\theta_S = 0$) and the domestic childrearing is intensive in female labor ($\omega_m < 1/2 < \omega_f$), fertility declines.

Proposition 1 shows that with no market childrearing services available, an increase in female market labor and structural change into service economy are both associated with a fertility decline. The next subsection sheds light on the market childrearing services.

3.3 General Equilibrium with Market Childrearing Services: $\theta_S > 0$

The model with market childrearing services is rather complex. Thus, we rely on numerical analysis to solve the model.

We conduct two types of numerical experiments. The first experiment assumes a constant θ_S . In the second experiment, we assume that θ_S increases over time, which means that the availability and the productivity of market childrearing services increase over time.

The first experiment uses the following parameter values: $L_m = L_f = 1$, $\xi = 0.5$, $\eta = 0.6$, $\zeta_C = 0.6$, $\epsilon = 2$, $\omega_f = 0.8$, $\gamma = 2$, $\theta_S = 0.1$, $\sigma = 5$, $\mu_{f,C} = 0.3$, and $\mu_{f,S} = 0.5$. It is assumed that A_C increases linearly with time from 1 to 10, while A_S increases linearly with time from 1 to 3. In the second experiment, we assume that θ_S increases linearly with time from 0.1 to 0.19.

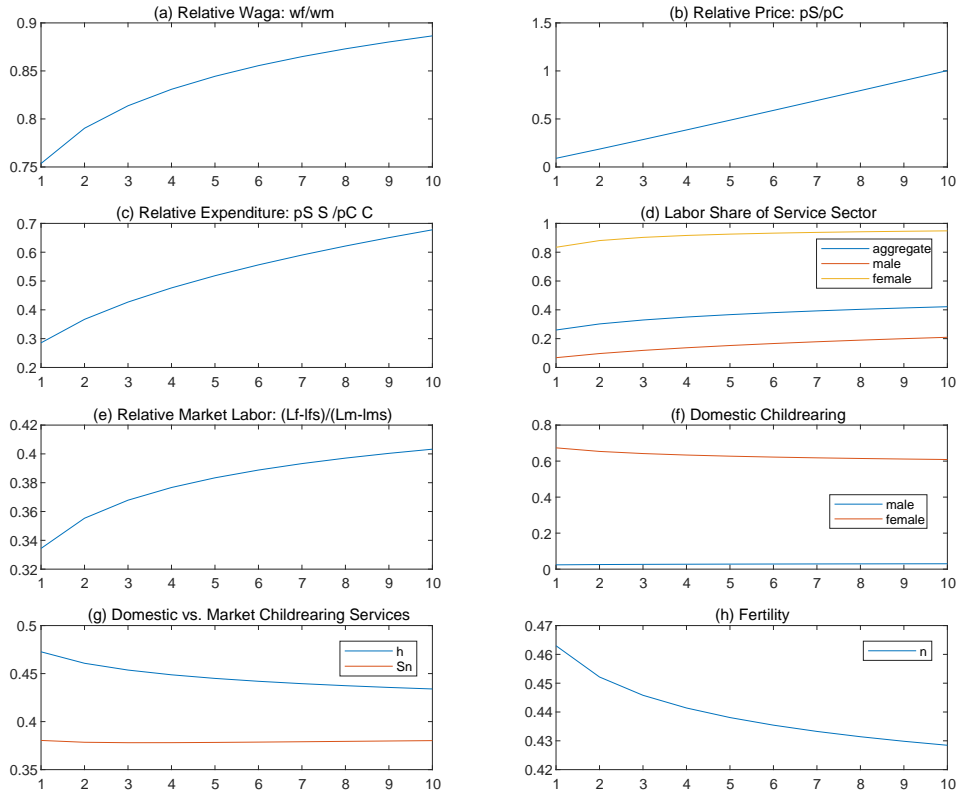


Figure 5: Equilibrium with a Constant θ_S

Figure 5 presents the results of the first experiment, while Figure 6 presents the results of the second experiment. If the availability and the productivity of market childrearing services remains unchanged (the first experiment), fertility declines as female market labor participation increases and female domestic childrearing time decreases (see Panels (e), (f), and (h) in Figure 5).

In contrast, if more market childrearing services become available over time (the second experiment), fertility follows the u-shaped dynamics. At the early stage of dynamics, fertility declines.

However, it increase over time at the late stage of dynamics, in spite of declining domestic childrearing (see Panels (f) and (h) in Figure 6). This fertility rise is accounted for mainly by the rise in the usage of market childrearing services, as shown in Panel (g) of Figure6.

Notice that if more market childrearing services become available over time, female market labor increases more and female domestic childrearing decrease more than they do if the availability and the productivity of market childrearing services remains unchanged (see Panels (e) and (f) in both Figures 5 and 6.

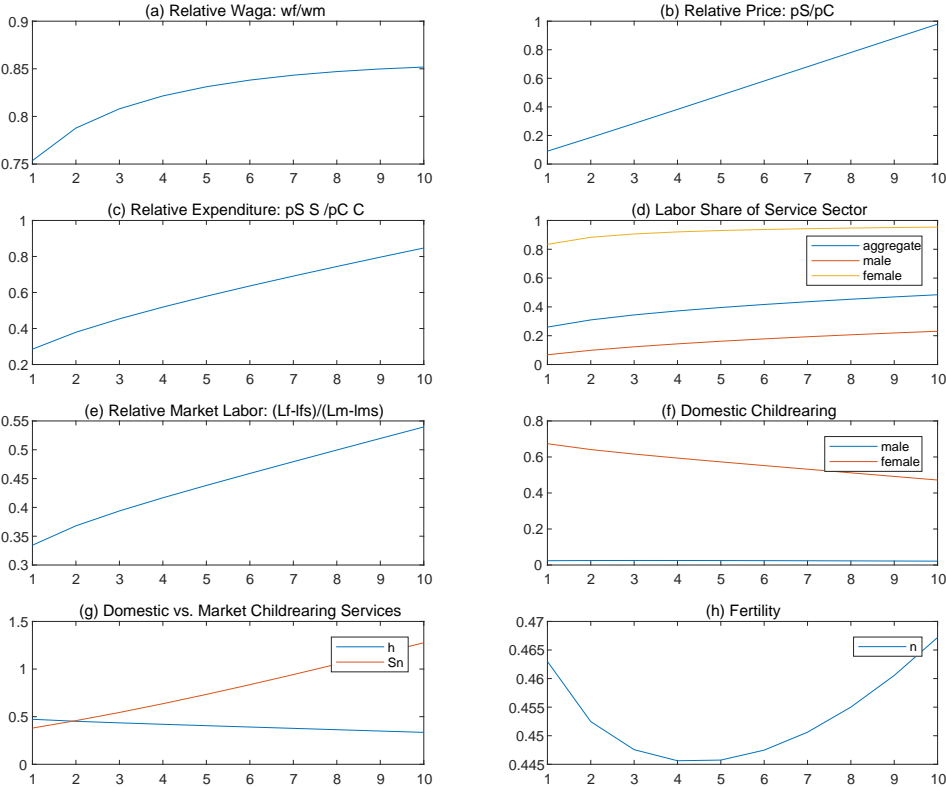


Figure 6: Equilibrium with a Variable θ_S

4 Conclusion

We show that the dynamics of fertility and female labor participation are closely associated with structural transformation toward service economy. These dynamics and interactions are significantly affected by the availability and productivity of market childrearing services.

Appendix

A Optimization

Utility Maximization of Households

We solve the utility maximization problem of households in two steps. The first step maximizes (2) subject to (6). Then, we can derive (8), (9), and (10). The second step maximizes (1) subject to (3), (4), and (7). The first-order conditions are given by

$$\frac{w_k}{P_D D} = \lambda h^{\frac{1}{\epsilon}} \omega_k l_k^{-\frac{1}{\epsilon}}, \quad k = m \text{ or } f, \quad (\text{A.1})$$

$$\frac{p_s}{P_D D} = \xi \theta_s n^{\frac{1-\gamma}{\gamma}} S_n^{-\frac{1}{\gamma}}, \quad (\text{A.2})$$

$$\lambda = \xi \theta_h n^{\frac{1-\gamma}{\gamma}} h^{-\frac{1}{\gamma}}, \quad (\text{A.3})$$

where λ is the costate variable associated with (4).

From (A.1), we obtain

$$l_m = \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^{\epsilon} l_f. \quad (\text{A.4})$$

Substituting (A.4) into (4) yields

$$h = l_f \left[\omega_m \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^{\epsilon-1} + \omega_f \right]^{\frac{\epsilon}{\epsilon-1}}. \quad (\text{A.5})$$

We use (A.4) and (A.5) to obtain

$$\begin{aligned} w_m l_m + w_f l_f &= \left[\left(\frac{\omega_m w_f}{\omega_f w_m} \right)^{\epsilon} w_m + w_f \right] l_f \\ &= \left[\omega_m \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^{\epsilon-1} + \omega_f \right] \frac{w_f}{\omega_f} l_f \\ &= \left[\frac{\omega_m^{\epsilon}}{w_m^{\epsilon-1}} + \frac{\omega_f^{\epsilon}}{w_f^{\epsilon-1}} \right]^{\frac{1}{1-\epsilon}} h. \end{aligned} \quad (\text{A.6})$$

Use (4), (A.1), and (A.3), we derive

$$\begin{aligned} \frac{w_m l_m + w_f l_f}{P_D D} &= \lambda h^{\frac{1}{\epsilon}} \left[\omega_m l_m^{\frac{\epsilon-1}{\epsilon}} + \omega_f l_f^{\frac{\epsilon-1}{\epsilon}} \right], \\ &= \lambda h \\ &= \xi \theta_h \left(\frac{h}{n} \right)^{\frac{\gamma-1}{\gamma}}. \end{aligned} \quad (\text{A.7})$$

Adding both sides of (A.2) and (A.7) and using (3), we obtain

$$w_m l_m + w_f l_f + p_s S_n = \xi P_D D. \quad (\text{A.8})$$

From (7) and (A.8), we obtain (11) and

$$w_m l_m + w_f l_f + p_s S_n = \frac{\xi}{1 + \xi} (w_m L_m + w_f L_f). \quad (\text{A.9})$$

(A.2) implies

$$\frac{p_S S_n}{P_D D} = \xi \theta_S \left(\frac{S_n}{n} \right)^{\frac{\gamma-1}{\gamma}}. \quad (\text{A.10})$$

From (3), (11), (A.9), and (A.10), we obtain

$$p_S S_n = \frac{\xi}{1 + \xi} (w_m L_m + w_f L_f) \frac{\theta_S S_n^{\frac{\gamma-1}{\gamma}}}{\theta_h h^{\frac{\gamma-1}{\gamma}} + \theta_S S_n^{\frac{\gamma-1}{\gamma}}}, \quad (\text{A.11})$$

$$w_m l_m + w_f l_f = \frac{\xi}{1 + \xi} (w_m L_m + w_f L_f) \frac{\theta_h h^{\frac{\gamma-1}{\gamma}}}{\theta_h h^{\frac{\gamma-1}{\gamma}} + \theta_S S_n^{\frac{\gamma-1}{\gamma}}}, \quad (\text{A.12})$$

$$\frac{p_S S_n}{w_m l_m + w_f l_f} = \frac{\theta_S}{\theta_h} \left(\frac{S_n}{h} \right)^{\frac{\gamma-1}{\gamma}}. \quad (\text{A.13})$$

We use (A.6) and (A.13) to obtain (12). If we use (12) in (A.11), we obtain (13). Then, (14) determines h . Solving (A.4) and (G.1), we determine l_f and l_m .

We rewrite (3) as

$$n = S_n \left[\theta_h H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}} + \theta_S \right]^{\frac{\gamma}{\gamma-1}}, \quad \gamma > 1 \text{ and } \theta_h + \theta_S = 1, \quad (\text{A.14})$$

We substitute (13) into the above equation and after some manipulation we obtain (17).

B Proof of Result 3

We rearrange (16) as

$$\frac{w_m l_m + w_f l_f}{w_m L_m + w_f L_f} = \frac{\xi}{1 + \xi} \frac{\frac{\theta_h}{\theta_S} H(p_S, w_m, w_f)^{\frac{\gamma-1}{\gamma}}}{1 + \frac{\theta_h}{\theta_S} H(p_S, w_m, w_f)^{\frac{\gamma-1}{\gamma}}}. \quad (\text{B.1})$$

Let us denote the right-hand side of this equation as $RHS(p_S, w_m w_f)$. Then, we have

$$\frac{w_m L_m L_f \left(\frac{l_f}{L_f} - \frac{l_m}{L_m} \right)}{(w_m L_m + w_f L_f)^2} + \frac{w_m \frac{\partial l_m}{\partial w_f} + w_f \frac{\partial l_f}{\partial w_f}}{w_m L_m + w_f L_f} = \frac{\partial RHS(p_S, w_m, w_f)}{\partial w_f}. \quad (\text{B.2})$$

Since $\gamma > 1$ and $H(p_S, w_m, w_f)$ decreases with w_f , $RHS(p_S, w_m w_f)$ also decrease with w_f . In addition, (15) implies

$$\frac{\partial l_m}{\partial w_f} = \epsilon \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{l_f}{w_f} + \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{\partial l_f}{\partial w_f}. \quad (\text{B.3})$$

From $l_f/L_f > l_m/L_m$, (B.2), and (B.3), we know that $\partial l_f/\partial w_f < 0$

We next examine the effect of p_S . From (B.1), we have

$$\frac{w_m \frac{\partial l_m}{\partial p_S} + w_f \frac{\partial l_f}{\partial p_S}}{w_m L_m + w_f L_f} = \frac{\partial RHS(p_S, w_m, w_f)}{\partial p_S}. \quad (\text{B.4})$$

In addition, (15) implies

$$\frac{\partial l_m}{\partial p_S} = \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{\partial l_f}{\partial p_S}. \quad (\text{B.5})$$

Since $RHS(p_S, w_m, w_f)$ is an increasing function of p_S , we have $\partial l_f / \partial p_S > 0$.

Finally, we examine the effect of w_m . We obtain from (B.2):

$$\frac{w_f L_m L_f \left(\frac{l_m}{L_m} - \frac{l_f}{L_f} \right)}{(w_m L_m + w_f L_f)^2} + \frac{w_m \frac{\partial l_m}{\partial w_m} + w_f \frac{\partial l_f}{\partial w_m}}{w_m L_m + w_f L_f} = \frac{\partial RHS(p_S, w_m, w_f)}{\partial w_m}. \quad (\text{B.6})$$

where the first term on the left-hand side and the right-hand side have negative signs. In addition, (15) implies

$$\frac{\partial l_m}{\partial w_m} = -\epsilon \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{l_f}{w_m} + \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{\partial l_f}{\partial w_m}. \quad (\text{B.7})$$

Then, if we solve (B.6) and (B.7), we obtain the expression for $\partial l_f / \partial w_m$, which has an ambiguous sign. \square

C Proof of Result 4

We take a logarithm of $(L_f - l_f)/(L_m - l_m)$ and then differentiate it with respect to w_f :

$$\begin{aligned} \frac{\partial}{\partial w_f} \ln \frac{L_f - l_f}{L_m - l_m} &= -\frac{\frac{\partial l_f}{\partial w_f}}{L_f - l_f} + \frac{\frac{\partial l_m}{\partial w_f}}{L_m - l_m} \\ &> -\frac{\frac{\partial l_f}{\partial w_f}}{L_f - l_f} + \frac{\left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon \frac{\partial l_f}{\partial w_f}}{L_m - l_m} \\ &= \frac{1}{L_m - l_m} \frac{\partial l_f}{\partial w_f} \left\{ \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^\epsilon - \frac{L_m - l_m}{L_f - l_f} \right\} \\ &> 0. \end{aligned}$$

The inequality in the second line holds because of (B.3). The last inequality holds because Result 3 ensures that $\partial l_f / \partial w_f < 0$ and (19) is assumed. \square

D Proof of Result 5

From (17) and (12), we obtain

$$\frac{\partial \ln n}{\partial w_f} = \frac{L_f}{w_m L_m + w_f L_f} - \frac{\left(\frac{\omega_f}{w_f} \right)^\epsilon \theta_h H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}}{\left(\theta_S + \theta_h H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}} \right) \left(\frac{\omega_m^\epsilon}{w_m^{\epsilon-1}} + \frac{\omega_f^\epsilon}{w_f^{\epsilon-1}} \right)}.$$

If other things equal, $\theta_h H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}$ increases with θ_h . Because of $\theta_S = 1 - \theta_h$, the second term in the right-hand side of the above equation monotonically increases with θ_h . Beside, the second term satisfies

$$\lim_{\theta_h \rightarrow 0} \text{Second term} = 0, \quad \text{and} \quad \lim_{\theta_h \rightarrow 1} \text{Second term} = \frac{\left(\frac{\omega_f}{w_f}\right)^\epsilon}{\frac{\omega_m^\epsilon}{w_m^{\epsilon-1}} + \frac{\omega_f^\epsilon}{w_f^{\epsilon-1}}}.$$

Note that the following relation holds:

$$\text{sign} \left\{ \frac{L_f}{w_m L_m + w_f L_f} - \frac{\left(\frac{\omega_f}{w_f}\right)^\epsilon}{\frac{\omega_m^\epsilon}{w_m^{\epsilon-1}} + \frac{\omega_f^\epsilon}{w_f^{\epsilon-1}}} \right\} = \text{sign} \left\{ \frac{L_f}{L_m} - \left(\frac{\omega_f w_m}{\omega_m w_f}\right)^\epsilon \right\}.$$

These facts imply Result 5. \square

E Proof of Result 7

From (26), we have

$$\frac{p_S}{p_C} = \frac{A_C}{A_S} \left[\left(\frac{\mu_{f,S}}{\mu_{f,C}}\right)^\sigma - \mu_{m,C}^\sigma \frac{\left(\frac{\mu_{f,S}}{\mu_{f,C}}\right)^\sigma - \left(\frac{\mu_{m,S}}{\mu_{m,C}}\right)^\sigma}{\mu_{m,C}^\sigma + \mu_{f,C}^\sigma \left(\frac{w_f}{w_m}\right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}. \quad (\text{E.1})$$

The inequality $\mu_{f,S} > \mu_{f,C}$ ensures that $\left(\frac{\mu_{f,S}}{\mu_{f,C}}\right)^\sigma - \left(\frac{\mu_{m,S}}{\mu_{m,C}}\right)^\sigma > 0$. Then, as long as $\sigma \neq 1$, p_S/p_C increases with w_f/w_m .

From (25), $y_C = C$, and $y_S = S (\equiv S_D + S_n)$, we have

$$\frac{L_{k,S}}{L_{k,C}} = \left(\frac{\mu_{k,S}}{\mu_{k,C}}\right)^\sigma \cdot \frac{p_S S}{p_C C} \cdot \frac{\mu_{m,C}^\sigma \left(\frac{w_f}{w_m}\right)^{\sigma-1} + \mu_{f,C}^\sigma}{\mu_{m,S}^\sigma \left(\frac{w_f}{w_m}\right)^{\sigma-1} + \mu_{f,S}^\sigma}. \quad (\text{E.2})$$

Clearly, $p_S S/p_C C$ has a positive effect on $L_{k,S}/L_{k,C}$. The inequality $\mu_{f,S} > \mu_{f,C}$ ensures that the right-hand side increases with $\left(w_f/w_m\right)^{\sigma-1}$ that increases with w_f/w_m if $\sigma > 1$. \square

F Labor Demand and Supply

We rewrite (25) as

$$L_{k,i} = \frac{p_i y_i}{w_k} \frac{\mu_{k,i}^\sigma \left(\frac{w_k}{w_m}\right)^{-\sigma}}{\mu_{m,i}^\sigma + \mu_{f,i}^\sigma \left(\frac{w_f}{w_m}\right)^{-\sigma}}, \quad (\text{F.1})$$

for $k = m$ or f and $i = C$ or S . We first consider $i = C$. If we use $y_C = C$, (8), (11), we have

$$\frac{p_C y_C}{w_k} = \frac{1}{1 + \xi} \left(\frac{w_m}{w_k} L_m + \frac{w_f}{w_k} L_f \right) \frac{\zeta_C^\eta}{\zeta_C^\eta + \zeta_S^\eta \left(\frac{p_S}{p_C} \right)^{1-\eta}}. \quad (\text{F.2})$$

Since p_S/p_C is a function of w_f/w_m (see (E.1)), $L_{k,C}$ ($k = m$ or f) also becomes a function of w_f/w_m .

We next consider $i = S$. Since $y_S = S_D + S_n$, we have

$$\frac{p_S y_S}{w_k} = \frac{p_S S_D}{w_k} + \frac{p_S S_n}{w_k}. \quad (\text{F.3})$$

Similarly to (F.2), $p_S S_D/w_k$ is a function of w_f/w_m as follows:

$$\frac{p_S S_D}{w_k} = \frac{1}{1 + \xi} \frac{w_m L_m + w_f L_f}{w_k} \frac{\zeta_S^\eta \left(\frac{p_S}{p_C} \right)^{1-\eta}}{\zeta_C^\eta + \zeta_S^\eta \left(\frac{p_S}{p_C} \right)^{1-\eta}}. \quad (\text{F.4})$$

(13) implies

$$\frac{p_S S_n}{w_k} = \frac{\xi}{1 + \xi} \left(\frac{w_m}{w_k} L_m + \frac{w_f}{w_k} L_f \right) \frac{1}{1 + \frac{\theta_h}{\theta_S} H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}}. \quad (\text{F.5})$$

We rewrite $H(p_S, w_m w_f)$ as

$$\begin{aligned} H(p_S, w_m w_f) &= \left(\frac{\theta_h p_S}{\theta_S w_f} \right)^\gamma \left[\omega_m^\epsilon \left(\frac{w_f}{w_m} \right)^{\epsilon-1} + \omega_f^\epsilon \right]^{\frac{\gamma}{\epsilon-1}} \\ &= \left(\frac{\theta_h}{\theta_S} \frac{1}{A_S} \left[\mu_{m,S}^\sigma \left(\frac{w_m}{w_f} \right)^{\epsilon-1} + \mu_{f,S}^\sigma \right]^{\frac{1}{1-\sigma}} \right)^\gamma \left[\omega_m^\epsilon \left(\frac{w_f}{w_m} \right)^{\epsilon-1} + \omega_f^\epsilon \right]^{\frac{\gamma}{\epsilon-1}}, \\ &\equiv H^* \left(\frac{w_f}{w_m} \right). \end{aligned} \quad (\text{F.6})$$

In the second equality, we use (26). Thus, from (F.1) and (F.3)–(F.6), $L_{k,S}$ is also a function of w_f/w_m .

(15) and (16) show that both l_m and l_f are functions of w_f/w_m . Thus, either (27) or (28) determines w_f/w_m . One of (27) and (28) is redundant.

In the numerical simulation, we use the following expression for l_f . From (A.5), we have

$$l_f = h \left[\omega_m \left(\frac{\omega_m w_f}{\omega_f w_m} \right)^{\epsilon-1} + \omega_f \right]^{-\frac{\epsilon}{\epsilon-1}},$$

where h is given by

$$\begin{aligned} h &= H^* \left(\frac{w_f}{w_m} \right) S_n, \\ &= H^* \left(\frac{w_f}{w_m} \right) \frac{\xi}{1 + \xi} \frac{w_f}{p_S} \left(\frac{w_m}{w_f} L_m + L_f \right) \frac{1}{1 + \frac{\theta_h}{\theta_S} H(p_S, w_m w_f)^{\frac{\gamma-1}{\gamma}}}. \end{aligned}$$

The first equality uses (14) while the second line uses (F.5).

G A Special Case: $\theta_S = 0$

To consider the market equilibrium, we rewrite (28) as $L_f - l_f = L_{f,C} + L_{f,S}$. Both the left- and right-hand sides are equations of w_f/w_m . We denote the left- and right-hand sides as $\Phi(x)$ and $\Psi(x)$, respectively, where $x = w_f/w_m$.

To characterize $\Phi(\cdot)$, we derive the closed form solution for l_f . Since $\theta_S = 0$, we have $S_n = 0$. Then, (16) can be rewritten as

$$w_m l_m + w_f l_f = \frac{\xi}{1 + \xi} (w_m L_m + w_f L_f). \quad (\text{G.1})$$

We substitute (A.4) into the above equation and then solve for l_f :

$$l_f = \frac{\xi}{1 + \xi} \frac{L_m + \frac{w_f}{w_f} L_f}{\left(\frac{\omega_m}{\omega_f} \frac{w_f}{w_m}\right)^\epsilon + \frac{w_f}{w_m}} = \frac{\xi}{1 + \xi} \left\{ L_f + \frac{L_m - \left(\frac{\omega_m}{\omega_f} \frac{w_f}{w_m}\right)^\epsilon L_f}{\left(\frac{\omega_m}{\omega_f} \frac{w_f}{w_m}\right)^\epsilon + \frac{w_f}{w_m}} \right\}. \quad (\text{G.2})$$

We assume that $L_m = L_f$ and $\omega_m < \omega_f$. As $w_m/w_f \rightarrow 0$, we have that $l_f \rightarrow +\infty$. As $w_m/w_f \rightarrow +\infty$, we have that (i) $l_f \rightarrow 0 (< L_f)$ if $\epsilon > 1$, (ii) $l_f \rightarrow \xi L_f / (1 + \xi) (< L_f)$ if $\epsilon < 1$, and (iii) $l_f \rightarrow \xi L_f / \{(1 + \xi)[1 + (\omega_m/\omega_f)^\epsilon]\} (< L_f)$ if $\epsilon = 1$. If $w_f/w_m = \omega_f/\omega_m$, we have that $l_f = \xi L_f / (1 + \xi) (< L_f)$.

$\Phi(\cdot)$ has the following properties. $\Phi(\cdot)$ satisfies that $\lim_{w_f/w_m \rightarrow 0} \Phi(\cdot) = -\infty$, $\lim_{w_f/w_m \rightarrow \omega_f/\omega_m} \Phi(\cdot) = L_f / (1 + \xi)$, and $\lim_{w_f/w_m \rightarrow +\infty} \Phi(\cdot) > 0$.

We next consider $\Psi(\cdot)$. Because of $y_S = S_D$, we have

$$\Psi\left(\frac{w_f}{w_m}\right) = \frac{1}{1 + \xi} \left(L_f + \frac{L_m}{\frac{w_f}{w_m}} \right) \frac{1}{\mu_{f,C}^\sigma + \mu_{m,C}^\sigma \left(\frac{w_f}{w_m}\right)^{\sigma-1}} \frac{\zeta_C^\eta \mu_{f,C}^\sigma + \zeta_S^\eta \mu_{f,S}^\sigma \left(\frac{A_C}{A_S}\right)^{1-\eta} \Gamma\left(\frac{w_f}{w_m}\right)}{\zeta_C^\eta + \zeta_S^\eta \left[\frac{A_C}{A_S} \Gamma\left(\frac{w_f}{w_m}\right)\right]^{1-\eta}},$$

where

$$\Gamma\left(\frac{w_f}{w_m}\right) = \left[\frac{\mu_{m,S}^\sigma + \mu_{f,S}^\sigma \left(\frac{w_f}{w_m}\right)^{1-\sigma}}{\mu_{m,C}^\sigma + \mu_{f,C}^\sigma \left(\frac{w_f}{w_m}\right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} \quad \text{and} \quad \sigma > 1 > \eta > 0.$$

Because of $\mu_{f,S} > \mu_{f,C}$, $\Gamma(\cdot)$ is an increasing function and satisfies $\lim_{w_f/w_m \rightarrow 0} \Gamma(\cdot) = (\mu_{f,C}/\mu_{f,S})^{\frac{\sigma}{\sigma-1}}$ and $\lim_{w_f/w_m \rightarrow +\infty} \Gamma(\cdot) = (\mu_{m,C}/\mu_{m,S})^{\frac{\sigma}{\sigma-1}}$. Then, we have that $\Psi(\cdot) > 0$, $\lim_{w_f/w_m \rightarrow 0} \Psi(\cdot) = +\infty$, and $\lim_{w_f/w_m \rightarrow +\infty} \Psi(\cdot) = 0$.

The discussion so far indicates that $\lim_{w_f/w_m \rightarrow 0} \Phi(\cdot) = -\infty < +\infty = \lim_{w_f/w_m \rightarrow 0} \Psi(\cdot)$ and $\lim_{w_f/w_m \rightarrow +\infty} \Phi(\cdot) > 0 = \lim_{w_f/w_m \rightarrow +\infty} \Psi(\cdot)$. Thus, $\Phi(\cdot)$ and $\Psi(\cdot)$ has at least one intersection. Therefore, at least one equilibrium exists. The following discussion assumes the uniqueness of the equilibrium, which is shown to be true for a wide range of parameters by our numerical analysis.

As a driving force of structural change, we consider an increase in A_C/A_S . We differentiate $\Psi(\cdot)$ with respect to A_C/A_S :

$$\text{sign} \frac{\partial \Psi}{\partial (A_C/A_S)} = \text{sign} \left\{ \left(\frac{\mu_{f,S}}{\mu_{f,C}} \right)^\sigma \Gamma(\cdot)^{\sigma-1} - 1 \right\}. \quad (\text{G.3})$$

Since $\Gamma(\cdot)$ is an increasing function that satisfies $\lim_{w_f/w_m \rightarrow 0} = (\mu_{f,C}/\mu_{f,S})^{\frac{\sigma}{\sigma-1}}$, we have

$$\left(\frac{\mu_{f,S}}{\mu_{f,C}}\right)^{\sigma} \Gamma(\cdot)^{\sigma-1} - 1 > \left(\frac{\mu_{f,S}}{\mu_{f,C}}\right)^{\sigma} \left(\frac{\mu_{f,C}}{\mu_{f,S}}\right)^{\sigma} - 1 = 0 \quad (\text{G.4})$$

for $w_f/w_m > 0$. An increase in A_C/A_S shifts $\Psi(\cdot)$ upward, moving the intersection between $\Phi(\cdot)$ and $\Psi(\cdot)$ rightward. Then, w_f/w_m increases (see Figure G.1).

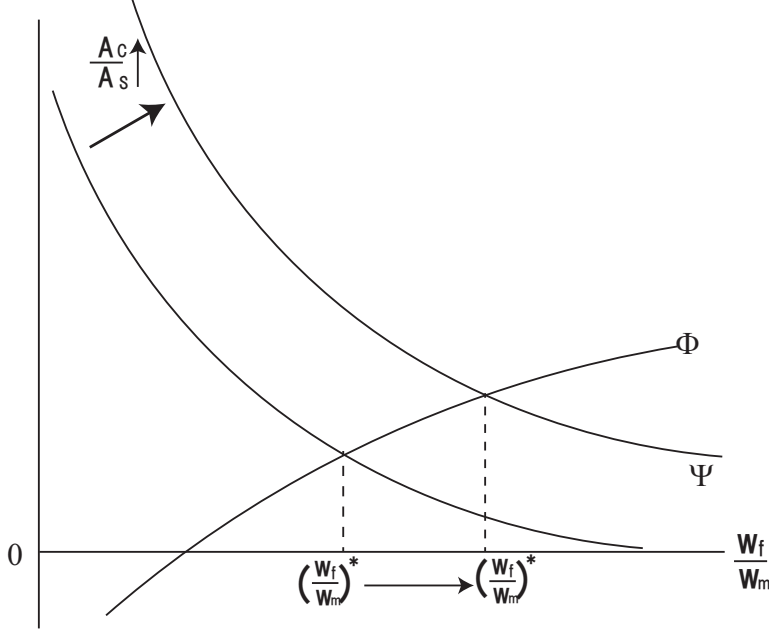


Figure G.1: Equilibrium and the Effect of Biased Technical Change

Since p_S/p_C increases with both w_f/w_m and A_C/A_C (see (E.1)), p_S/p_C also increases. Result 1 indicates that $p_S S/(p_C C)$ increases (see (18)). Result 7-2 indicates that $L_{k,S}/L_{k,C}$ increases (see (E.2)).

Because of $L_m = L_f$, (G.2) shows that if $w_f/w_m < 1 (< \omega_f/\omega_m)$, l_f decreases with w_f/w_m . If (19) holds, Result 4 shows that $(L_f - l_f)/(L_m - l_m)$ increases. Please see Appendix C and note that proof of Result 4 holds even when we differentiate $\ln(L_f - l_f)/(L_m - l_m)$ with respect to w_f/w_m .

We finally examine the effect on fertility. When $\theta_S = 0$, we have $n = h$. We substitute (G.2) into (A.5) and after some manipulation we obtain

$$n = \frac{\xi}{1 + \xi} \left(\frac{L_m}{w_f/w_m} + L_f \right) \left\{ \omega_m^\epsilon \left(\frac{w_f}{w_m} \right)^{\epsilon-1} + \omega_f^\epsilon \right\}^{\frac{1}{\epsilon-1}}. \quad (\text{G.5})$$

We take a logarithm of n and differentiate it with respect to w_f/w_m :

$$\text{sign} \frac{\partial \ln n}{\partial w_f/w_m} = \text{sign} \left\{ \frac{L_f}{L_m} - \left(\frac{\omega_f w_m}{\omega_m w_f} \right)^\epsilon \right\}. \quad (\text{G.6})$$

Then, if (20) holds, fertility decreases.

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