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Monetary Policy, Financial Frictions, and Heterogeneous R&D Firms in an Endogenous Growth Model*

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Abstract

Motivated by the empirical facts, we construct an endogenous growth model in which heterogeneous R&D firms are financially constrained and use cash to finance R&D investments. The interaction between financial constraints and heterogeneity is an important determinant of the optimality of the Friedman rule. If there are no financial frictions, the presence of heterogeneity does not affect the optimality of the Friedman rule. If there are severe financial frictions, however, heterogeneity has an important effect. Without heterogeneity, the Friedman rule is optimal. However, with heterogeneity among R&D firms, deviating from the Friedman rule improves social welfare under a plausible condition.

Keywords: R&D, heterogeneous firms, financial frictions, cash-in-advance constraint, the Friedman rule

JEL classification numbers: E52, O30, O40

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1 Introduction

One of the fundamental issues in macroeconomics is how monetary policy should be set in both the long run and the short run. Thus, many authors study the optimality of the Friedman rule, which sets the nominal interest rate to zero. The empirical facts show that firms rely on cash to finance research and development (R&D) investments. In addition, many authors argue that external finance for R&D may be more costly than other types of investments (see Section 1.2). Since the interest rate affects the opportunity cost of holding cash and firms' access to external funds, examining monetary policy in the context of R&D investments is important. Nevertheless, studies of the Friedman rule and R&D are limited. This study examines the optimality of the Friedman rule in an R&D-based endogenous growth model. We argue that the interaction between financial frictions and firm heterogeneity is important for the optimality of the Friedman rule.

The following three facts motivate us: (a) financial constraints matter for R&D investments more than for other types of investments, (b) firms with high R&D ability are more likely to be financially constrained, and (c) firms tend to rely on cash to finance R&D. Section 1.2 discusses these facts in detail.

To reflect these facts, we extend the quality-ladder model proposed by Grossman and Helpman (1991). R&D activities improve the productivity of intermediate goods production. Based on fact (b), we introduce heterogeneity in R&D productivity among R&D firms. At each moment in time, R&D firms decide whether to conduct R&D activities. R&D firms with productivity higher (lower) than a threshold operate (do not operate). As in Grossman and Helpman (1991), R&D firms need external funds to finance R&D investments. To capture fact (a), we depart from the standard setting as follows. After completing R&D activities, R&D firms decide whether to repay external funds. Defaulting firms are not caught with a positive probability. Thus, R&D firms have an incentive to default. Because of this financial friction, R&D firms' access to external funds is limited. According to fact (c), we assume that R&D firms need cash to finance R&D because of a cash-in-advance (CIA) constraint, as in Chu and Cozzi (2014). Thus, monetary policy affects firm-level and aggregate R&D investments; an increase in the nominal interest rate raises the opportunity cost of holding cash and then depresses R&D. Although we build on the quality-ladder model as a benchmark, our main results hold in a variety-expansion model. To emphasize the importance of financial frictions and heterogeneity, we also consider cases without them.

Our main results are summarized as follows. First, if there are no financial constraints (R&D firms' access to external funds is not limited), the condition for the optimality of the Friedman rule does not qualitatively depend on the presence of heterogeneity in R&D productivity.[1] Second, however, if there are severe financial constraints (R&D firms' access to external funds is severely limited), the presence of heterogeneity in R&D productivity affects the optimality of the Friedman rule. Without heterogeneity, the Friedman rule is optimal. If there is heterogeneity, the Friedman rule is not optimal under a plausible condition.

The intuition behind the first result is simple. Suppose that there are heterogeneity in R&D productivity and no financial constraints. Then, through competition among R&D firms, the most productive R&D firms capture all of the funds for R&D and only these firms operate. Thus, heterogeneity among R&D firms does not matter.

The intuition behind the second result is as follows. Consider the case without heterogeneity. Severe financial constraints limit firms' access to funds for R&D and hence R&D intensity

[1] The qualitatively same condition may provide quantitatively different results because the values of the calibrated parameters of the models with and without heterogeneity are generally different from each other.

becomes inefficiently low. Decreasing the nominal interest rate to zero reduces the cost of R&D and then stimulates R&D activities, which makes R&D intensity closer to the optimal level. Thus, the Friedman rule is optimal.

Next, consider heterogeneity in R&D productivity among R&D firms. Financial constraints generate inefficient resource allocations in the following two ways. First, the resource allocation within the R&D sector is distorted. Because of severe financial constraints, the most productive R&D firms cannot capture all of the funds for R&D and then they employ an inefficiently small amount of resources. By contrast, low-productivity R&D firms employ an inefficiently large amount of resources. Thus, aggregate R&D productivity becomes low. Second, financial constraints affect the resource allocation between the R&D sector and intermediate goods sector. Since the aggregate productivity of the R&D sector is low, the resources allocated to the R&D sector tend to be inefficiently large. An increase in the nominal interest rate raises the cost of R&D and makes low-productivity R&D firms inoperative, which improves the resource allocation within the R&D sector and thus increases aggregate R&D productivity. In addition, the resources allocated to the R&D sector are reduced. Thus, deviating from the Friedman rule improves social welfare.

The numerical analysis shows that there is a unique optimal nominal interest rate under plausible parameter values. Without heterogeneity, the optimal nominal interest rate decreases as financial constraints become severer. With heterogeneity, we obtain the opposite result: the optimal nominal interest rate increases as financial constraints become severer. This result is counterintuitive. Since severe financial constraints depress R&D investments, one may conjecture that the monetary authority should decrease the nominal interest rate to counteract the negative effect of financial constraints on R&D investments. This applies to the homogeneous case. However, this conjecture is not true for the heterogeneous case. As financial constraints become severer, low-productivity R&D firms become operative and hence the within-R&D sector distortion becomes larger. Aggregate R&D productivity becomes lower and the resources allocated to the R&D sector become inefficiently larger, which generates a larger between-sector distortion. An increase in the nominal interest rate mitigates these distortions by making low-productivity R&D firms inoperative and reducing the resources allocated to the R&D sector. Thus, the optimal nominal interest rate increases as financial constraints become severer.

Our results indicate that the interaction between financial frictions and heterogeneity is important for monetary policies. Moreover, this study suggests that ignoring financial frictions and heterogeneity may provide inappropriate guidance for monetary policies.

1.1 Relation to the Literature

Many authors have studied monetary policy in various macroeconomic settings. Chari and Kehoe (1999) show that the Friedman rule is optimal in the long run, using standard monetary models (the CIA model with credit goods, the money-in-the-utility (MIU) model, and the shopping-time model).[2] However, only a handful of theoretical works investigate R&D activities and growth despite their empirical relevance to monetary policy. We review the theoretical literature on monetary policy, confining our attention to the R&D-based growth literature.

Marquis and Reffett (1994) introduce a CIA constraint on consumption goods in a variety-expansion model and show that positive nominal interest rates generate welfare losses. Chu and Lai (2013) combine the MIU-function and quality-ladder models. They numerically show that welfare is maximized when the nominal interest rate is zero. These authors' results are

[2]Mulligan and Sala-I-Martin (1997) examine the optimality of the Friedman rule by using a framework that includes a wide range of models as special cases.

in favor of the Friedman rule. Funk and Kromen (2010) also construct a quality-ladder model with the MIU function, emphasizing the roles of sticky price. However, they do not conduct a welfare analysis. Oikawa and Ueda (2015) examine the optimal inflation rate in a cashless economy with R&D activities and menu costs. Their calibrated model shows that the optimal inflation rate is close to the growth-maximizing inflation rate, -2%. These studies make unique contributions. However, none of them reflects the fact that cash is used for R&D investments. The channels through which monetary policy affects R&D in these studies are different from those in our model. In addition, these authors do not consider financial constraints and heterogeneity in the R&D sector, both of which are empirically relevant to monetary policy and are shown to be important determinants of the optimality of the Friedman rule in our model.

Berentsen et al. (2012) construct a search-theoretic model in which cash is needed for R&D investments. Their simulation shows that reducing inflation to zero has sizable welfare gains. Chu and Cozzi (2014) construct a quality-ladder model in which R&D firms face a CIA constraint.[3] They show that the Friedman rule is optimal if the equilibrium is characterized by underinvestment, while it is not optimal if the equilibrium is characterized by overinvestment. Both studies assume that agents borrow cash to conduct R&D and that their borrowings are *not* limited. Neither study considers heterogeneous R&D productivity. These authors shed light on the fact that firms rely on cash to finance R&D and then provide useful R&D-based growth models suitable for the analysis of monetary policy. Nonetheless, they do not consider financial constraints and heterogeneity in the R&D sector. This study complements the literature by constructing a tractable R&D-based growth model that highlights the empirical facts: firms use cash to finance R&D investments and heterogeneity in R&D productivity matters in the presence of financial frictions. Constructing such a model is important because we show that models with and without financial constraints and heterogeneity in the R&D sector provide opposite policy implications.

1.2 Empirical Facts on R&D and Financial Constraints

Since Arrow (1962), many authors have discussed why financial frictions matter, particularly for investments in innovation. Asymmetric information related to R&D is one of the sources of financial frictions. Since R&D investments tend to require more technical knowledge than other types of investments, firms conducting R&D often have much better information about the characteristics of their own R&D projects than do outside investors. In addition, the intangible nature of the assets being produced by R&D is also the source of financial frictions. Compared with R&D investments, physical investments are more likely to produce tangible assets that can be used as collateral. Then, external financing for R&D tends to be more costly than physical investments.

Many authors empirically examine whether financial constraints affect R&D activities. Early studies were based on the idea that if firms are financially constrained (i.e., their access to external funds is limited), changes in the available internal funds including cash flow affect their investments or R&D expenditure. By using US firm data, Himmelberg and Petersen (1994) find a strong positive relationship between R&D and internal funds. Brown et al. (2009) provide evidence that the increase in US corporate cash flow in the 1990s stimulated R&D in that period. By using Compustat data, Brown and Petersen (2009) show that for 1970–2006, physical investments became less sensitive to cash flow, while R&D remained strongly sensitive to cash flow, which suggests that financial constraints matter for R&D more than physical investments. Mulkay et al. (2001) and Bond et al. (2006) find that financial constraints on

[3] We extend the model of Chu and Cozzi (2014). Thus, their model is included in our model as a special case.

R&D are stronger for US and UK firms than firms in other European countries. However, Brown et al. (2012) point out the possibility of the downward bias of these results for European countries. Indeed, they find that if they control for the liquidity management of firms, financial constraints do matter for the R&D of European firms.

These studies suggest that financial frictions affect the R&D investments of firms. However, they are based on indirect measures of financial constraints such as changes in internal funds. Recent studies have started to shift focus onto more direct measures of financial constraints. Suppose that firms are asked to imagine that they receive additional cash exogenously and decide how to spend it. Then, we can directly observe whether firms choose to invest the cash in additional R&D projects. If a firm decides to invest in additional R&D projects, the R&D investments of the firm are financially constrained.

By using direct measures of financial constraints, several authors examine their effects on R&D investments. Hajivassiliou and Savignac (2008) find that financial constraints have a significantly negative effect on the innovation of French firms. Hottenrott and Peters (2012) use data on manufacturing firms in Germany and show that financial constraints hold back innovation activities. By using data on countries in Eastern Europe and the Commonwealth of Independent States, Gorodnichenko and Schnitzer (2013) find that financial constraints restrain the innovation activities of domestically owned firms.

The studies mentioned thus far provide direct or indirect evidence that financial constraints affect R&D investments significantly. Monetary policy affects firms' access to external funds through interest rate control. Thus, studying monetary policy along with R&D investments is important. Interestingly, Hajivassiliou and Savignac (2008) and Hottenrott and Peters (2012) find that firms with high innovation ability are more likely to be financially constrained. This finding shows the importance of studying heterogeneous innovation ability among firms, along with financial frictions.

While the above empirical studies show that financial frictions affect R&D investments, the mechanisms behind financial frictions are not identified. In this study, we simply assume that because defaulting R&D firms are not caught with a positive probability, the borrowings of R&D firms are limited. This assumption simplifies the analysis and allows us to focus on the interaction between financial frictions and heterogeneity.

We now turn to the facts on the cash holdings of R&D firms. As pointed out by Falato and Sim (2014), the top cash holders in the United States are innovative corporates that conduct R&D investments heavily. By using US data on non-financial firms, Falato and Sim (2014) show that firms with positive R&D expenditure hold a relatively large proportion of their assets in the form of cash because external financing for R&D is costly. Bates et al. (2009) find that from 1980 to 2006, the cash holdings of US industrial firms were negatively correlated with physical investment and that firms with higher R&D expenditures hold more cash. By using panel data for US manufacturing firms over 1970–2006, Brown and Petersen (2011) find that firms most likely to face financial constraints rely extensively on cash holdings to smooth R&D expenditure. Brown et al. (2012) report similar evidence for European firms. Berentsen et al. (2012) and Chu and Cozzi (2014) also provide empirical reviews that emphasize the importance of cash for R&D to motivate their models.

Since firms tend to rely on cash to finance R&D, monetary policies may affect the R&D activities of firms through their effects on the opportunity cost of holding cash. By using inflation as a proxy for the cost of holding cash, Pinkowitz et al. (2003) and Ramírez and Tadesse (2009) provide evidence that inflation has a negative effect on the cash holdings of firms, although the effect is statistically insignificant for some of their regression specifications. Evers et al. (2009) use US data and find that higher inflation restrains firm-level R&D investment because it reduces corporate cash holdings.

The discussion thus far motivates us to construct an R&D-based endogenous growth model in which heterogeneous R&D firms facing financial constraints use cash to finance R&D. Our goal is to explore the effects of monetary policy on R&D activities and growth, focusing on financial constraints and heterogeneity. For simplicity, as in Berentsen et al. (2012) and Chu and Cozzi (2014), we assume that R&D firms borrow cash at a rental rate equal to the nominal interest rate in the equilibrium. This approach can be justified by the above facts that regardless of the ways in which R&D firms obtain their financing, they rely on cash extensively to finance R&D and thus monetary policy affects R&D activities.

2 The Model

Our model is based on the quality-ladder growth model of Grossman and Helpman (1991). Chu and Cozzi (2014) introduce a CIA constraint into the model of Grossman and Helpman. We introduce financial frictions and heterogeneity in R&D firms into the model of Chu and Cozzi (2014). Section 7 shows that our main results also hold in a variety-expansion growth model.

Time is continuous and denoted by $t \geq 0$. There is a continuum of intermediate goods whose measure is one. A single final good is produced by using a continuum of intermediate goods. Each intermediate good is produced by a monopolistically competitive firm. R&D firms conduct R&D activities to improve the productivity of intermediate goods production. As in Chu and Cozzi (2014), we remove the scale effects. The population size of the economy is $N_t = e^{nt}$, where n is the exogenous population growth rate. Since the quality-ladder model is well known, we describe the standard features of the model briefly.

2.1 Final Good

The technology of the final good production is given by

$$Y_t = A_t \cdot exp\left\{\int_0^1 \ln(x_{j,t})dj\right\}, \tag{1}$$

where $x_{j,t}$ is the input of intermediate good $j \in [0,1]$. A_t captures Comin's (2004) argument that R&D investments contribute to a small proportion of productivity growth. Exogenous growth rate $g_A \equiv \dot{A}_t/A_t$ represents productivity growth, which is accounted for by factors other than R&D. We denote the price of intermediate good j in terms of the final good as $p_{j,t}$. The demand function for intermediate good j is

$$x_{j,t} = Y_t/p_{j,t}. \tag{2}$$

2.2 Intermediate Goods

We assume that there are no financial frictions and no CIA constraints in the intermediate goods sector as in Chu and Cozzi (2014). Section 6 observes that the financial frictions and CIA constraints in the intermediate goods sector do not affect our results.

Each intermediate good is produced by an industry leader until the arrival of the next innovation. The production technology of intermediate good j at time t is

$$x_{j,t} = z^{s_{j,t}} l_{x,j,t}, \quad z > 1, \tag{3}$$

where $l_{x,j,t}$ is the labor input and $z^{s_{j,t}}$ is labor productivity. $s_{j,t}$ takes a positive integer that represents the number of innovations that have occurred in industry j until time t.

Standard price competition leads to a profit-maximizing price. We consider patent breadth similar to Goh and Olivier (2002) and Chu and Cozzi (2014). We assume that the maximum markup that intermediate goods firms can set is $\psi > 1$, where ψ is a policy instrument determined by the patent authority. Since the demand function (2) has unit price elasticity, firm j can maximize its operating profits by setting $p_{j,t} = \psi w_t / z^{s_{j,t}}$. Then, we obtain

$$l_{x,j,t} = \frac{Y_t}{\psi w_t} \equiv l_{x,t} \quad \left(\text{or} \quad x_{j,t} = \frac{z^{s_{j,t}} Y_t}{\psi w_t}\right), \tag{4a}$$

$$\pi_{x,j,t} = \frac{\psi - 1}{\psi} Y_t \equiv \pi_{x,t}, \tag{4b}$$

where $\pi_{x,j,t}$ is the operating profits of firm j. We denote the value of intermediate goods firm j in terms of the final good as $q_{j,t}$. Since all firms earn the same level of operating profits, $q_{j,t}$ becomes independent of j, $q_{j,t} = q_t$, and q_t satisfies

$$r_t q_t = \pi_{x,t} - \iota_t q_t + \dot{q}_t, \tag{5}$$

where ι_t is the aggregate R&D intensity derived in the next subsection.

For later use, we derive the following equations by using (1), (3), and (4a):

$$w_t = A_t Z_t / \psi, \quad \text{and} \quad Y_t = A_t Z_t l_{x,t}, \tag{6}$$

where $Z_t \equiv exp\left\{\int_0^1 (\ln z^{s_{j,t}}) dj\right\}$ grows through R&D investment.

2.3 R&D Sector

There is a continuum of R&D firms whose measure is one. Each R&D firm is owned by households. We denote the R&D productivity of an R&D firm as φ. Consider an infinitesimal short time interval of length dt. As every time interval dt passes, the R&D productivity of each R&D firm changes. At time t, each R&D firm draws productivity from a distribution, $F(\varphi)$. At time $t + dt$, it draws productivity again from the same distribution. φ is independent and identically distributed (iid) across both time and firms. The maximum (minimum) value of φ is $\varphi_{max} \geq 0$ ($\varphi_{min} \geq 0$). If $\varphi_{min} = \varphi_{max}$, $F(\varphi)$ is a degenerate distribution function. If $0 \leq \varphi_{min} < \varphi_{max}$, $F(\varphi)$ is a continuously differentiable distribution function with $F'(\cdot) > 0$. Later, we take the limit of $dt \to 0$ to obtain a continuous time model.[4]

Firms with the same R&D productivity level behave symmetrically. Consider R&D firms that draw productivity of level φ at time t. Between t and $t + dt$, these firms behave as follows. First, firms observe the value of φ and then determine the amount of labor, $l_{R,\varphi,t} \cdot dt$, that they employ. At the same time, firms determine the amount of real money, $m_{R,\varphi,t} \cdot dt (= M_{R,\varphi,t}/P_t \cdot dt)$, that they borrow from households, where $M_{R,\varphi,t}$ is nominal money and P_t is the price of the final good. The rental rate of money is $i_t \geq 0$. Since firms use money to pay labor wages, the following CIA constraint must be satisfied:

$$w_t l_{R,\varphi,t} \cdot dt \leq m_{R,\varphi,t} \cdot dt, \tag{7}$$

[4] Moll (2014) considers an iid shock in a continuous-time neoclassical growth model.

where w_t is the labor wage rate in terms of the final good. Since R&D firms cannot repay $m_{R,\varphi,t} \cdot dt$ to households until they earn revenue from R&D production, households are effectively providing credit to R&D firms. If an R&D firm decides not to conduct R&D ($l_{R,\varphi,t} = 0$), it does nothing until time $t + dt$.

Second, R&D firms conduct R&D. The intermediate goods that an R&D firm targets are selected randomly. However, R&D firms can choose the number of intermediate goods that they target. Given labor employment $l_{R,\varphi,t} \cdot dt$, an R&D firm with productivity φ can target $\iota_{\varphi,t} \cdot dt$ units of intermediate goods, where $\iota_{\varphi,t} \cdot dt$ is given by

$$\iota_{\varphi,t} \cdot dt = B\varphi \frac{l_{R,\varphi,t}}{N_t} \cdot dt, \quad B > 0. \tag{8}$$

The presence of the population size, N_t, captures the dilution effect that removes scale effects as in Laincz and Peretto (2006) and Chu and Cozzi (2014). Since dt is infinitesimally short and there are an infinite number of intermediate goods, more than an innovation does not happen to the same intermediate good in the interval of dt. The blueprints of newly improved intermediate goods are sold to households. The value of these blueprints in terms of the final good is q_t. An R&D firm earns revenue of $q_t \iota_{\varphi,t} \cdot dt$.

Finally, R&D firms decide whether to repay money including rental cost $(1 + i_t)m_{R,\varphi,t} \cdot dt$, where $i_t \geq 0$ is the rental rate of money. As in Liu and Wang (2014), we assume that if an R&D firm defaults, it is caught with probability $\theta_R \cdot dt$ ($\theta_R > 0$).[5] In this case, its revenue is seized and the firm is permanently excluded from future access to credit. If a defaulting R&D firm is not caught with probability $1 - \theta_R \cdot dt$, households cannot distinguish the firm from non-defaulting firms. Thus, the firm retains future access to credit.

Financial Constraints. We derive an incentive constraint for R&D firms that ensures that no default occurs in the equilibrium. Denote the values of non-defaulting and defaulting firms with productivity φ at time t in terms of the final good as $v^N_{\varphi,t}$ and $v^D_{\varphi,t}$, respectively. Define $v_{\varphi,t} \equiv max\{v^N_{\varphi,t},\ v^D_{\varphi,t}\}$. The expected value of $v_{\varphi,t+dt}$ is $v_{t+dt} \equiv \int v_{\varphi,t+dt} dF(\varphi)$. Then, $v^N_{\varphi,t}$ is given by

$$v^N_{\varphi,t} = \left[q_t \iota_{\varphi,t} + (m_{R,\varphi,t} - w_t l_{R,\varphi,t}) - (1 + i_t)m_{R,\varphi,t}\right] dt + \frac{v_{t+dt}}{1 + r_t \cdot dt}, \tag{9}$$

where r_t is the real interest rate. The term $m_{R,\varphi,t} - w_t l_{R,\varphi,t}$ is the remaining real money after the labor wage payment. The term $(1 + i_t)m_{R,\varphi,t}$ is the repayment of money including the rental cost. Since the productivity of R&D firms changes at time $t + dt$, v_{t+dt} appears in (9). Since defaulting firms do not repay $(1 + i_t)m_{R,\varphi,t}$ and are not caught with probability $1 - \theta_R \cdot dt$, $v^D_{\varphi,t}$ is given by

$$\begin{aligned} v^D_{\varphi,t} &= \left[q_t \iota_{\varphi,t} + (m_{R,\varphi,t} - w_t l_{R,\varphi,t})\right](1 - \theta_R \cdot dt) \cdot dt + \frac{1 - \theta_R \cdot dt}{1 + r_t \cdot dt} v_{t+dt} \\ &= \left[q_t \iota_{\varphi,t} + (m_{R,\varphi,t} - w_t l_{R,\varphi,t})\right] dt + \frac{1 - \theta_R \cdot dt}{1 + r_t \cdot dt} v_{t+dt}. \end{aligned}$$

The second line uses $(dt)^2 \approx 0$. If $v^N_{\varphi,t} \geq v^D_{\varphi,t}$, R&D firms have no incentive to default. By using

[5] Liu and Wang (2014) consider a discrete-time model, while we consider continuous time.

$(dt)^2 \approx 0$, we rearrange $v^N_{\varphi,t} \geq v^D_{\varphi,t}$ as

$$(1 + i_t)m_{R,\varphi,t} \leq \theta_R v_{t+dt}. \tag{10}$$

Optimization. As far as (10) is satisfied, we have $v_{\varphi,t} \equiv max\{v^N_{\varphi,t},\ v^D_{\varphi,t}\} = v^N_{\varphi,t}$. R&D firms choose $l_{R,\varphi,t}$ and $m_{R,\varphi,t}$ to maximize $v_{\varphi,t}$ subject to (7), (8), and (10). We solve this problem in two steps. First, R&D firms choose $m_{R,\varphi,t}$ given $l_{R,\varphi,t}$:

$$v_{\varphi,t} = \max_{m_{R,\varphi,t}} \left\{ \left[q_t \iota_{\varphi,t} + (m_{R,\varphi,t} - w_t l_{R,\varphi,t}) - (1 + i_t)m_{R,\varphi,t} \right] dt + \frac{v_{t+dt}}{1 + r_t \cdot dt} \quad \text{s.t. (7)} \right\}.$$

If $i_t > 0$, the solution to this problem is $m_{R,\varphi,t} = w_t l_{R,\varphi,t}$. We can rewrite (10) as

$$(1 + i_t)w_t l_{R,\varphi,t} \leq \theta_R v_{t+dt}. \tag{11}$$

Second, R&D firms choose $l_{R,\varphi,t}$ subject to $w_t l_{R,\varphi,t} = m_{R,\varphi,t}$, (8), and (11), which is formulated as

$$v_{\varphi,t} = \max_{l_{R,\varphi,t}} \left\{ \left[\frac{q_t B\varphi}{N_t} - (1 + i_t)w_t \right] l_{R,\varphi,t} \cdot dt + \frac{v_{t+dt}}{1 + r_t \cdot dt} \quad \text{s.t. (11)} \right\}. \tag{12}$$

This problem gives the next solution:

$$l_{R,\varphi,t} = \begin{cases} 0, & \text{if } \varphi < \underline{\varphi}_t, \\ \frac{\theta_R v_{t+dt}}{(1+i_t)w_t}, & \text{if } \varphi \geq \underline{\varphi}_t, \end{cases} \quad \text{where } \underline{\varphi}_t \equiv \frac{(1 + i_t)w_t N_t}{Bq_t}. \tag{13}$$

To derive (13), we assume $i_t > 0$. When $i_t = 0$, CIA constraint (7) may not bind and (13) may not hold. When $i_t = 0$, R&D firms maximize $v_{\varphi,t}$ subject to $w_t l_{R,\varphi,t} \leq m_{R,\varphi,t} \leq \theta_R v_{t+dt}$. If $\varphi \geq \underline{\varphi}_t|_{i_t=0}$, firms can maximize $v_{\varphi,t}$ by choosing $w_t l_{R,\varphi,t} = m_{R,\varphi,t} = \theta_R v_{t+dt}$. If $\varphi < \underline{\varphi}_t|_{i_t=0}$, R&D firms choose $l_{R,\varphi,t} = 0$. Thus, even if $i_t = 0$, (13) still holds. However, the choice of $m_{R,\varphi,t}$ is indeterminate. For simplicity, we assume that R&D firms choose $m_{R,\varphi,t} = 0$ if $\varphi < \underline{\varphi}_t|_{i_t=0}$.

Substituting (13) into (8) yields

$$\iota_{\varphi,t} = \begin{cases} 0, & \text{if } \varphi < \underline{\varphi}_t, \\ \frac{B\varphi\theta_R v_{t+dt}}{(1+i_t)w_t N_t}, & \text{if } \varphi \geq \underline{\varphi}_t, \end{cases} \quad \text{where } \underline{\varphi}_t \equiv \frac{(1 + i_t)w_t N_t}{Bq_t}. \tag{14}$$

Firms with high productivity ($\varphi \geq \underline{\varphi}_t$) conduct R&D. All of these firms conduct the same level of R&D because φ is an iid shock. When θ_R is large, active R&D firms can employ much labor and conduct R&D intensively. An increase in i_t raises the cost of R&D, which negatively affects the $\iota_{\varphi,t}$ of active R&D firms and the number of active R&D firms, $1 - F(\underline{\varphi}_t)$.

Define $\pi_{R,\varphi,t} \equiv \max\left\{0, \varphi/\underline{\varphi}_t - 1\right\}$ and $\pi_{R,t} \equiv \int \pi_{R,\varphi,t} dF(\varphi) = \int_{\underline{\varphi}_t}^{\infty} \left(\varphi/\underline{\varphi}_t - 1\right) dF(\varphi)$. From (12) and (13), $v_{\varphi,t}$ satisfies $v_{\varphi,t} = \pi_{R,\varphi,t}\theta_R v_{t+dt}dt + \frac{v_{t+dt}}{1+r_t \cdot dt}$. Aggregating this equation over φ yields $v_t = \pi_{R,t}\theta_R v_{t+dt}dt + \frac{v_{t+dt}}{1+r_t \cdot dt}$. We rearrange this equation and take the limit of $dt \to 0$ to obtain

$$r_t v_t = \pi_{R,t}\theta_R v_t + \dot{v}_t. \tag{15}$$

We aggregate (13) and (14) and take the limit of $dt \to 0$:

$$l_{R,t} = \int l_{R,\varphi,t} dF(\varphi) = \frac{\theta_R v_t (1 - F(\underline{\varphi}_t))}{(1+i_t)w_t}, \tag{16}$$

$$\iota_t = \int \iota_{\varphi,t} dF(\varphi) = \frac{B\theta_R v_t}{(1+i_t)w_t N_t} \int_{\underline{\varphi}_t}^{\infty} \varphi dF(\varphi). \tag{17}$$

From these two equations, we obtain

$$\iota_t = B\Gamma(\underline{\varphi}_t)\frac{l_{R,t}}{N_t}, \quad \text{where } \Gamma(\underline{\varphi}_t) \equiv \frac{\int_{\underline{\varphi}_t}^{\infty} \varphi dF(\varphi)}{1 - F(\underline{\varphi}_t)}. \tag{18}$$

$\Gamma(\underline{\varphi}_t)$ is the average productivity of active R&D firms that represents the aggregate productivity of the R&D sector. If $F(\varphi)$ is a non-degenerate distribution, we have $\Gamma'(\underline{\varphi}_t) > 0$ for $\underline{\varphi}_t \in (\varphi_{min}, \varphi_{max})$ and $\lim_{\underline{\varphi}_t \to \varphi_{max}} \Gamma(\underline{\varphi}_t) = \varphi_{max}$.[6] If $F(\varphi)$ is a degenerate distribution, we have $\Gamma(\underline{\varphi}_t) = \varphi$ if $\varphi > \underline{\varphi}_t$.

When there is no financial friction ($\theta_R = +\infty$), only R&D firms with the highest productivity conduct R&D.[7] This means that heterogeneous productivity among R&D firms does not matter. Our model corresponds to that studied in Chu and Cozzi (2014).

2.4 Households

Consider a representative household whose size is $N_t = e^{nt}$. The utility of the representative household is

$$U_t = \int_t^{\infty} \frac{{c_s}^{1-\sigma}}{1-\sigma} e^{-\rho(s-t)} ds, \quad \rho > 0, \quad \sigma > 1, \tag{19}$$

where c_s is the per capita consumption of the final good at time s, ρ is the subjective discount rate, and $1/\sigma$ is the intertemporal elasticity of substitution. We restrict our attention to the case of $\sigma > 1$ because empirical studies find that the intertemporal elasticity of substitution is small.

At time t, the representative household holds $N_t m_t$ units of real money and lends $N_t b_t$ units of money to R&D firms at the rental rate of i_t. The number of intermediate goods firms that the representative household owns is $N_t \eta_t$. At time t, the household purchases $N_t \omega_t$ units of (newly invented) intermediate goods firms at the price of q_t. Therefore, b_t, m_t, η_t, and ω_t must satisfy

$$b_t \leq m_t \quad \text{and} \quad \dot{\eta}_t = \omega_t - (n + \iota_t)\eta_t. \tag{20}$$

The budget constraint in per capita terms is given by

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + w_t - c_t - (\mu_t + n)m_t + i_t b_t - q_t \omega_t + \eta_t \pi_{x,t} + \frac{\pi_{R,t}\theta_R v_t}{N_t} + T_t, \tag{21}$$

where a_t is per capita asset holdings other than money, $\mu_t \equiv \dot{P}_t / P_t$ is the inflation rate (P_t is the final good price), and T_t is the lump-sum transfer from the government. $\eta_t \pi_{x,t}$ and $\pi_{R,t}\theta_R v_t / N_t$

[6] We have $\text{sign}\{\Gamma'(\underline{\varphi})\} = -\underline{\varphi} + \Gamma(\underline{\varphi}) > 0$.

[7] Suppose that an R&D firm with productivity less than φ_{max} is active. Then, R&D firms with φ_{max} earn positive profits. With $\theta_R = +\infty$, the labor demand of R&D firms with φ_{max} is infinite, which cannot happen in the equilibrium.

are the profit incomes from intermediate goods firms and R&D firms, respectively.

The representative household maximizes (19) subject to (20) and (21). After rearranging the first-order conditions, we can derive the usual Euler equation:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho - n). \tag{22}$$

In addition, we obtain the same equation as (5) and $i_t = r_t + \mu_t$. The latter equation shows that the rental rate of money becomes equal to the nominal interest rate in the equilibrium.

2.5 Monetary Authority

The monetary authority controls nominal interest rate i_t, which is kept constant over time ($i_t = i \geq 0$). It rebates seigniorage revenue to households through lump-sum transfers. Then, $T_t = \dot{M}_t/(N_t P_t)$ holds, where M_t is the nominal money stock at time t.

2.6 Equilibrium Conditions

The equilibrium in the asset market requires $a_t = 0$. The representative household as a whole holds $N_t m_t$ units of real money and R&D firms borrow $\int m_{R,\varphi,t} dF(\varphi)(= w_t l_{R,t})$ units of money. Then, $N_t m_t = w_t l_{R,t}$ must hold. The number of intermediate goods firms that the representative household owns is equal to one, $N_t \eta_t = 1$. From $N_t \eta_t = 1$ and the second equation of (20), we can derive $N_t \omega_t = \iota_t$. The final good market clears as $c_t N_t = Y_t$. The equilibrium condition for the labor market is $l_{x,t} + l_{R,t} = N_t$. If we use (18), this equation can be rewritten as

$$\frac{l_{x,t}}{N_t} = 1 - \frac{\iota_t}{B\Gamma(\underline{\varphi}_t)}. \tag{23}$$

3 Steady State Equilibrium

This section derives a steady state equilibrium, where ι_t and $\underline{\varphi}_t$ are constant over time. From (18) and (23), $l_{R,t}/N_t$ and $l_{x,t}/N_t$ also become constant over time in the steady state equilibrium.

We define $Q_t \equiv A_t Z_t N_t / q_t$ and $V_t \equiv A_t Z_t N_t / v_t$. Both Q_t and V_t become constant at the steady state equilibrium. The following discussion omits time index t from the variables that are constant over time at a steady state. The steady state values of ι, $\underline{\varphi}$, and Q are determined by

$$\underline{\varphi} = \frac{1+i}{B\psi} Q, \tag{24a}$$

$$\iota = \frac{\theta_R \int_{\underline{\varphi}}^{\infty} \left(\frac{\varphi}{\underline{\varphi}} - 1 \right) dF(\varphi) - \rho - (\sigma - 1) g_A}{(\sigma - 1) \ln z}, \tag{24b}$$

$$\iota = \frac{\frac{\psi - 1}{\psi} Q - \rho - (\sigma - 1) g_A}{(\sigma - 1) \ln z + 1 + \frac{\psi - 1}{\psi} \frac{Q}{B\Gamma(\underline{\varphi})}}. \tag{24c}$$

Appendix A derives these three equations. Once the values of $\underline{\varphi}$, ι, and Q are determined, the other endogenous variables are determined as follows: (18) determines $l_{R,t}/N_t$. From (17), we have $V = \psi B \theta_R / [(1+i)\iota] \int_{\underline{\varphi}}^{\infty} \varphi dF(\varphi)$, which determines the steady state value of V. The real interest rate is given by $r = \rho + n + \sigma[g_A + \iota \ln z]$, where $\iota \ln z \equiv g_Z$ is the growth rate of Z_t (see (A.1) in Appendix A). The next equation determines $l_{x,t}/N_t$:

$$\frac{\psi - 1}{\psi} Q \frac{l_{x,t}}{N_t} = \rho + (\sigma - 1)g_A + [(\sigma - 1)\ln z + 1]\iota. \tag{25}$$

Appendix A derives (25). The second equation of (6) indicates that per capita output grows at a rate of $g = g_A + \iota \ln z (= g_A + g_Z)$. Since per capita consumption also grows at g, (22) and $i = r + \mu$ show that the inflation rate is given by $\mu = i - \rho - n - \sigma g$. Because of $m_t = w_t l_{R,t}/N_t$ and the first equation of (6), the growth rate of nominal money, M_t, is equal to $g + \mu + n$.

We next examine the existence of the steady state equilibrium and conduct some comparative statics. We first consider the homogeneous productivity case.

3.1 Homogeneous Productivity in the R&D Sector

If $F(\varphi)$ is a degenerate distribution such that $\varphi_{min} = \varphi_{max} \equiv \varphi$, then we have $\Gamma(\underline{\varphi}_t) = \varphi$ if $\varphi > \underline{\varphi}_t$. As far as $\varphi > \underline{\varphi}$, (24a)-(24c) can be summarized as

$$\iota = \frac{\theta_R \left(\frac{\psi}{1+i} \frac{B\varphi}{Q} - 1 \right) - \rho - (\sigma - 1)g_A}{(\sigma - 1)\ln z}, \tag{26a}$$

$$\iota = \frac{\frac{\psi - 1}{\psi} Q - \rho - (\sigma - 1)g_A}{(\sigma - 1)\ln z + 1 + \frac{\psi - 1}{\psi}\frac{Q}{B\varphi}}. \tag{26b}$$

We define

$$Q_a(\theta_R, i) \equiv \frac{\psi B\varphi}{1 + i} \frac{\theta_R}{\rho + (\sigma - 1)g_A + \theta_R} \quad \text{and} \quad Q_b \equiv \frac{\psi}{\psi - 1}[\rho + (\sigma - 1)g_A].$$

The right-hand sides of (26a) and (26b) become equal to zero when $Q = Q_a(\theta_R, i)$ and $Q = Q_b$, respectively. By using (26a) and (26b), Appendix B proves the next proposition.

Proposition 1. *Suppose that $\sigma > 1$ and that $F(\varphi)$ is a degenerate distribution. If $\lim_{\theta_R \to +\infty} Q_a(\theta_R, i) > Q_b$, there exists a unique $\underline{\theta_R}$ that satisfies $Q_a(\underline{\theta_R}, i) = Q_b$ given $i \geq 0$. If θ_R is larger than $\underline{\theta_R}$, there exists a unique steady state equilibrium, where $\iota > 0$ holds and we have*

$$(i)\ \frac{\partial \iota}{\partial i} < 0. \tag{27a}$$

$$(ii)\ \frac{\partial \iota}{\partial \theta_R} > 0 \ \textit{and} \ \lim_{\theta_R \to \underline{\theta_R}} \iota = 0. \tag{27b}$$

$$(iii)\ \frac{\partial \Gamma(\underline{\varphi})}{\partial i} = \frac{\partial \Gamma(\underline{\varphi})}{\partial \theta_R} = 0. \tag{27c}$$

Proposition 1 suggests that if the financial constraint is not too severe, positive growth is possible. An increase in i raises the cost of R&D and then reduces ι. With a larger θ_R, R&D firms can employ more labor, which positively affects ι. Finally, if R&D firms are homogeneous, monetary policy does not affect the aggregate productivity of the R&D sector.

3.2 Heterogeneous Productivity in the R&D Sector

If $F(\varphi)$ is a non-degenerate distribution, (24b) still holds. From (24a) and (24c), we obtain

$$\iota = \frac{\frac{\psi-1}{1+i}B\underline{\varphi} - \rho - (\sigma-1)g_A}{(\sigma-1)\ln z + 1 + \frac{\psi-1}{1+i}\frac{\underline{\varphi}}{\Gamma(\underline{\varphi})}}. \tag{28}$$

By using (24b) and (28), we derive the steady state equilibrium. We define $\underline{\varphi}_a(\theta_R)$ by

$$\theta_R \int_{\underline{\varphi}_a(\theta_R)}^{\infty} \left(\frac{\varphi}{\underline{\varphi}_a(\theta_R)} - 1 \right) dF(\varphi) = \rho + (\sigma-1)g_A. \tag{29}$$

The right-hand side of (24b) becomes equal to zero when $\underline{\varphi} = \underline{\varphi}_a(\theta_R)$. We have $\lim_{\theta_R \to +\infty} \underline{\varphi}_a(\theta_R) = \varphi_{max}$. Totally differentiating the above equation yields

$$\frac{d\underline{\varphi}_a(\theta_R)}{d\theta_R} = \frac{\rho + (\sigma-1)g_A}{\theta_R^2 \int_{\underline{\varphi}_a(\theta_R)}^{\infty} \frac{\varphi}{[\underline{\varphi}_a(\theta_R)]^2} dF(\varphi)} > 0.$$

Thus, $\underline{\varphi}_a(\theta_R)$ is an increasing function. In addition, we define

$$\underline{\varphi}_b \equiv \frac{1+i}{(\psi-1)B}[\rho + (\sigma-1)g_A].$$

Note that $\underline{\varphi}_b$ depends on i. The right-hand side of (28) becomes equal to zero when $\underline{\varphi} = \underline{\varphi}_b$. By using (24b) and (28), Appendix C proves the next proposition.

Proposition 2. *Suppose that $\sigma > 1$ and that $F(\varphi)$ is a non-degenerate distribution such that $0 < \varphi_{min} < \underline{\varphi}_b < \varphi_{max}$. Given i, there exists a unique $\underline{\theta_R}^H$ that satisfies $\underline{\varphi}_a(\underline{\theta_R}^H) = \underline{\varphi}_b$. Suppose that θ_R is larger than $\underline{\theta_R}^H$. If $\theta_R < +\infty$, or if $\theta_R = +\infty$ and $\varphi_{max} < +\infty$, there exists a unique steady state equilibrium, where $\iota > 0$ holds and we have*

$$(i)\ \frac{\partial \iota}{\partial i} < 0 \ \ and \ \ \frac{\partial \underline{\varphi}}{\partial i} \begin{cases} > 0, & if \ \theta_R < +\infty, \\ = 0, & if \ \theta_R = +\infty. \end{cases} \tag{30a}$$

$$(ii)\ \frac{\partial \iota}{\partial \theta_R} > 0, \ \frac{\partial \underline{\varphi}}{\partial \theta_R} > 0, \ and \ \lim_{\theta_R \to \underline{\theta_R}^H} \underline{\varphi} = \underline{\varphi}_b. \tag{30b}$$

$$(iii)\ \frac{\partial \Gamma(\underline{\varphi})}{\partial i} \begin{cases} > 0, & if \ \theta_R < +\infty, \\ = 0, & if \ \theta_R = +\infty, \end{cases} \quad and \quad \frac{\partial \Gamma(\underline{\varphi})}{\partial \theta_R} > 0. \tag{30c}$$

Similar to the homogeneous case, if the financial constraint is not too severe, positive growth is possible. An increase in i reduces ι, while an increase in θ_R raises ι. Both i and θ_R affect $\underline{\varphi}$. When i increases, the cost of R&D increases and hence R&D firms with low productivity cannot conduct R&D activities. Thus, $\underline{\varphi}$ increases. When θ_R increases, high-productive R&D firms increase the amount of labor they employ (see (14)), which stimulates labor demand and hence raises w_t. Since the cost of R&D increases, $\underline{\varphi}$ increases.

In contrast to the homogeneous case, monetary policy affects the aggregate productivity of the R&D sector. Suppose that $\theta_R < +\infty$. Increases in i and θ_R make firms with low productivity inoperative. Hence, the aggregate productivity of the R&D sector increases. If $\theta_R = +\infty$, only the most productive R&D firms conduct R&D activities, $\underline{\varphi} = \varphi_{max}$. Hence, an increase in i does not affect the aggregate productivity of the R&D sector.

4 Welfare

This section examines the optimality of the Friedman rule. Per capita output grows at a rate of $g = g_A + \iota \ln z (= g_A + g_Z)$. If we use $c_t N_t = Y_t$, the second equation of (6), and (23), social welfare is rewritten as

$$U = \frac{\left\{A_0 Z_0 \left(1 - \frac{l_{R,t}}{N_t}\right)\right\}^{1-\sigma}}{1-\sigma} \frac{1}{(\sigma - 1)(g_A + \iota \ln z) + \rho},$$

where A_0 and Z_0 are the initial values of A_t and Z_t, respectively. By using $l_{R,t}/N_t = \iota/(B\Gamma(\underline{\varphi}))$, we differentiate U with respect to i:

$$\text{sign}\left\{\frac{\partial U}{\partial i}\right\} = \left(L_R - \frac{l_{R,t}}{N_t}\right) B\Gamma(\underline{\varphi})\sigma(\ln z)\frac{\partial \iota}{\partial i} + [(\sigma - 1)(g_A + \iota \ln z) + \rho]\frac{\iota}{\Gamma(\underline{\varphi})}\frac{\partial \Gamma(\underline{\varphi})}{\partial i}, \tag{31}$$

where

$$L_R \equiv \frac{B\Gamma(\underline{\varphi}) \ln z - \rho - (\sigma - 1)g_A}{B\Gamma(\underline{\varphi})\sigma \ln z}. \tag{32}$$

The nominal interest rate affects social welfare through its effects on (a) the labor allocation within the R&D sector and (b) the labor allocation between the intermediate goods and R&D sectors. The second term in (31) represents the *within-sector effect*. Consider the heterogeneous case. An increase in the nominal interest rate makes low-productivity R&D firms inoperative. Labor employment in the R&D sector concentrates on high-productive R&D firms and aggregate R&D productivity, $\Gamma(\underline{\varphi})$, increases. This improves social welfare. If the productivity of R&D firms is homogeneous, this effect does not work because $\Gamma(\underline{\varphi})$ is constant at φ.

The first term in (31) represents the *between-sector effect*. An increase in i, raising the cost of R&D, shifts labor employment from the R&D sector to the intermediate goods sector. Then, $l_{R,t}/N_t$ decreases. If $l_{R,t}/N_t > L_R$, the between-sector effect becomes positive because we have $\partial\iota/\partial i < 0$ (see Propositions 1 and 2). An increase in i has a positive welfare effect. Thus, inequality $l_{R,t}/N_t > L_R$ suggests that labor employment in the R&D sector is inefficiently large, while that in the intermediate goods sector is inefficiently small. Note that L_R increases with $B\Gamma(\underline{\varphi})$, which means that when the aggregate productivity of the R&D sector is higher, more labor should be allocated to the R&D sector. The following two points are important: (i) if there is no heterogeneity in R&D firms' productivity, L_R becomes constant because of $\Gamma(\underline{\varphi}) = \varphi$ and (ii) if there is heterogeneity among R&D firms, Proposition 2 indicates that L_R increases with θ_R and i. Thus, the between-sector effect depends on financial frictions and monetary policy.

To examine the optimality of the Friedman rule, the remainder of this section mainly evaluates the sign of $\partial U/\partial i$ at $i = 0$.

4.1 No Financial Frictions: $\theta_R = +\infty$

If there is no heterogeneity in R&D firms' productivity, the sign of $\partial U/\partial i|_{i=0,\ \theta_R=+\infty}$ is equal to the first term of (31). In (31), L_R is constant because $\Gamma(\underline{\varphi}) = \varphi$ holds without heterogeneity.

To consider the case where there is heterogeneity in R&D firms' productivity, this subsection assumes $\varphi_{max} < +\infty$, which ensures the existence of the steady state (see Proposition 2). If there is no financial friction ($\theta_R = +\infty$), we have $\Gamma(\underline{\varphi}) = \varphi_{max}$. Thus, the within-sector effect vanishes. In addition, L_R becomes constant because of $\Gamma(\underline{\varphi}) = \varphi_{max}$. Similar to the homogeneous case, the between-sector effect determines the sign of $\partial U/\partial i|_{i=0,\ \theta_R=+\infty}$.

Therefore, without financial frictions, if $L_R < l_{R,t}/N_t$, we have $\partial U/\partial i|_{i=0,\ \theta_R=+\infty} > 0$, which means that the Friedman rule is not optimal, while if $L_R > l_{R,t}/N_t$, the Friedman rule is optimal. This result does not depend on the presence of heterogeneity in R&D firms' productivity. In this sense, without financial frictions, heterogeneity in R&D firms' productivity does not affect the optimality of the Friedman rule.[8]

4.2 Homogeneity and Financial Frictions: $\varphi_{min} = \varphi_{max} \equiv \varphi$ and $\theta_R < +\infty$

Without heterogeneity, we have $\Gamma(\underline{\varphi}) = \varphi$. Thus, L_R is constant. We prove the next proposition.

Proposition 3. *Suppose that Proposition 1 holds and that $L_R > 0$ and $i = 0$. If $\theta_R(> \underline{\theta_R}|_{i=0})$ is sufficiently close to $\underline{\theta_R}|_{i=0}$ that is defined in Proposition 1, we have $\partial U/\partial i|_{i=0} < 0$. Moreover,*
(i) if $\partial U/\partial i|_{i=0,\ \theta_R=+\infty} < 0$, we have $\partial U/\partial i|_{i=0} < 0$ for all $\theta_R(> \underline{\theta_R}|_{i=0})$.
(ii) if $\partial U/\partial i|_{i=0,\ \theta_R=+\infty} > 0$, there exists a unique $\overline{\theta_R}(> \underline{\theta_R}|_{i=0})$ such that we have $\partial U/\partial i|_{i=0} < 0$ for $\theta_R \in (\underline{\theta_R}|_{i=0}, \overline{\theta_R})$, while we have $\partial U/\partial i|_{i=0} > 0$ for $\theta_R > \overline{\theta_R}$.

(Proof) Since ι decreases as θ_R decreases (see the first inequality of (27b)), $l_{R,t}/N_t(= \iota/(B\varphi))$ also decreases as θ_R decreases. From the second equation of (27b) and $l_{R,t}/N_t = \iota/(B\varphi)$, we have $\lim_{\theta_R \to \underline{\theta_R}} l_{R,t}/N_t = 0$. If $L_R > l_{R,t}/N_t$ holds for $\theta_R = +\infty$, then $L_R > l_{R,t}/N_t$ holds for all $\theta_R(> \underline{\theta_R})$. If $L_R < l_{R,t}/N_t$ holds for $\theta_R = +\infty$, there exists a unique $\overline{\theta_R}(> \underline{\theta_R})$ such that $L_R < (>) l_{R,t}/N_t$ for $\theta_R > (<) \overline{\theta_R}$. Since the second term in (31) is absent, Proposition 3 is derived. □

If there is no heterogeneity in R&D productivity, severe financial frictions make the Friedman rule optimal. The intuition is simple. Since severe financial frictions limit R&D firms' access to external funds, R&D firms employ only a small amount of labor ($l_{R,t}/N_t < L_R$). An increase in the nominal interest rate decreases the labor allocated in the R&D sector further and thus social welfare deteriorates. Hence, the Friedman rule is optimal. Proposition 3 (i) shows that if the Friedman rule is optimal without financial frictions, it is also optimal in the presence of financial frictions. Proposition 3 (ii) shows that even if the Friedman rule is not optimal without financial frictions, severe financial frictions make the Friedman rule optimal.

The next proposition examines the characteristics of the optimal nominal interest rate.

Proposition 4. *Suppose that Proposition 3 (ii) holds and that $\theta_R > \overline{\theta_R}$ holds, which implies $\partial U/\partial i|_{i=0} > 0$. Then, there exists a unique $i^* > 0$ that maximizes U and increases with θ_R.*

(Proof) Since $\theta_R > \overline{\theta_R}$, we have $L_R < l_{R,t}/N_t$ for $i = 0$. Because of $l_{R,t}/N_t = \iota/(B\varphi)$, $l_{R,t}/N_t$ monotonically decreases with i (see (27a)). Define $\bar{i}$ by $Q_a(\theta_R, \bar{i}) = Q_b$. The proof of Proposition 1 implies that as i approaches $\bar{i}$, $l_{R,t}/N_t(= \iota/(B\varphi))$ approaches zero. Then, there exists a unique $i^* \in (0, \bar{i})$ such that $L_R = l_{R,t}/N_t(= \iota/(B\varphi))$ and i^* maximizes U. Since ι increases with θ_R, i^* must increase with θ_R for $L_R = l_{R,t}/N_t(= \iota/(B\varphi))$ to hold. □

As θ_R increases beyond $\overline{\theta_R}$, more and excessive labor is allocated to R&D. To achieve the optimal allocation, the monetary authority should thus increase i^* to depress R&D.

[8] If $\varphi = \varphi_{max}$, where φ is R&D productivity in the homogeneous case and φ_{max} is the highest productivity in the heterogeneous case, the two economies are identical if there are no financial frictions.

4.3 Heterogeneity and Financial Frictions: $\varphi_{min} < \varphi_{max}$ and $\theta_R < +\infty$

When there is heterogeneity in R&D firms' productivity, we can prove the next proposition.

Proposition 5. *Suppose that Proposition 2 holds and that the following inequality holds:*

$$B \cdot \Gamma\left(\underline{\varphi}_b|_{i=0}\right) \cdot \ln z < \rho + (\sigma - 1)g_A, \tag{33}$$

and $i = 0$. Then, if $\theta_R(> \underline{\theta_R}^H|_{i=0})$ is sufficiently close to $\underline{\theta_R}^H|_{i=0}$, we have $\partial U/\partial i|_{i=0} > 0$. Here, $\underline{\theta_R}^H$ and $\underline{\varphi}_b$ are defined in Section 3.2.

(Proof) As shown in (30b), we have $\lim_{\theta_R \to \underline{\theta_R}^H} \underline{\varphi} = \underline{\varphi}_b$. The inequality, (33), implies that $L_R < 0$ when $\theta_R(> \underline{\theta_R}^H)$ is sufficiently close to $\underline{\theta_R}^H$. Proposition 2 ensures that $\iota > 0$ and hence $l_{R,t}/N_t = \iota/(B\Gamma(\underline{\varphi})) > 0$. Thus, we have $L_R < l_{R,t}/N_t$, which implies that the between-sector effect is positive when $\theta_R(> \underline{\theta_R}^H)$ is sufficiently close to $\underline{\theta_R}^H$. Proposition 2 shows that the within-sector effect is also positive. Then, we have $\partial U/\partial i|_{i=0} > 0$. □

With heterogeneity in R&D firms' productivity, severe financial frictions may make the Friedman rule undesirable. This result is in sharp contrast to that obtained in the absence of heterogeneity among R&D firms.

Condition (33) ensures that for sufficiently small θ_R, the between-sector effect is positive. The intuition behind this result is as follows. When financial frictions are severe, R&D firms with low productivity conduct R&D and thus the aggregate productivity of the R&D sector becomes low. As a result, L_R becomes small and thus labor employment in the R&D sector becomes inefficiently large ($l_{R,t}/N_t > L_R$). If the monetary authority increases i, labor is reallocated from the R&D sector to the intermediate goods sector, which improves social welfare. In addition, the within-sector effect works in the presence of heterogeneity. If i increases, low-productivity R&D firms become inoperative and hence the aggregate productivity of the R&D sector increases. This also improves social welfare. Thus, the Friedman rule is not optimal in an economy with severe financial frictions and heterogeneity in R&D firms.

Importantly, Proposition 5 holds even if the Friedman rule is optimal without financial constraints. This is counterintuitive. If the Friedman rule is optimal, an increase in the nominal interest rate decreases social welfare and reduces ι. This suggests that when the Friedman rule is optimal, ι may be inefficiently low (underinvestment in R&D).[9] A smaller θ_R depresses ι further (see Section 3). Thus, one may conjecture that if the Friedman rule is optimal without financial constraints, it is also optimal under severe financial constraints. This applies to the homogeneous case (see Proposition 3 (i)). However, this conjecture is not true in the presence of heterogeneity. Indeed, the next section shows numerically that under plausible parameters, the Friedman rule is not optimal if financial constraints are severe, while it is optimal if financial constraints are loose.

We mention the following two points. First, even when the between-sector effect is negative, the Friedman rule becomes undesirable if the positive within-sector effect is sufficiently strong. Thus, (33) is the sufficient condition for $\partial U/\partial i|_{i=0} > 0$. Second, (33) implies $L_R < 0$ for θ_R sufficiently close to $\underline{\theta_R}^H$. This does not mean that $L_R < 0$ holds for all $\theta_R(> \underline{\theta_R}^H)$, because L_R increases with θ_R. In fact, the next section numerically provides the cases where the between-sector effect is positive for small θ_R, while it is negative for large θ_R.

Before closing this section, we examine whether (33) holds under plausible parameters.

[9] Chu and Cozzi (2014) show that in their model without heterogeneity and financial frictions, the optimality of the Friedman rule is related to underinvestment in R&D.

Following Liu and Wang (2014), We assume that φ follows a Pareto distribution, $F(\varphi) = 1 - (\varphi_{min}/\varphi)^a$, where $a > 1$ and $\varphi_{min} > 0$ ($\varphi_{max} = +\infty$). Then, we have $\Gamma(\underline{\varphi}) = \frac{a}{a-1}\underline{\varphi}$. After some manipulation, we can rewrite (33) as

$$a[(\psi - 1) - \ln z] > \psi - 1.$$

We follow Acemoglu and Akcigit (2014) to set $z = 1.05$. We set $\psi = 1.225$, which corresponds to the intermediate value of the empirical estimates reported in and Jones and Williams (2000). Chu and Cozzi (2014) also use these values. Then, the above condition becomes

$$a > 1.103... \tag{34}$$

Since $a > 1$, (33) holds for a wide range of a under the plausible values of ψ and z.

5 Numerical Analysis

Section 4 analytically shows that monetary policy may have different welfare effects depending on financial frictions and heterogeneity. Section 4 focusing on cases with no financial frictions and extremely severe financial frictions. The numerical analyses in this section examine the optimality of the Friedman rule under mild and plausible levels of financial frictions. In addition, we numerically study the optimal nominal interest rate. Our purpose is not to examine the exact value of the optimal nominal interest rate because our model is too simple to identify it. Instead, we focus on the qualitative effects of financial frictions on the optimal nominal interest rate.

5.1 Calibration

As in Section 4, we set $z = 1.05$ and $\psi = 1.225$. In addition, we set $\sigma = 2$ and $\rho = 0.05$. These are conventional values in the growth literature.

The values of g_A, θ_R, and B as well as the parameters concerning the distribution of φ are determined as follows. Since the long-run growth rate of per capita GDP in the United States is about 2%, we choose the parameters so that $g = g_A + \iota \ln z = g_A + g_Z = 0.02$. Comin (2004) shows that of the 2% long-run growth, only about 0.2% may be due to R&D investment, while Chu (2010) finds that about 0.8% may be driven by domestic R&D in the United States. As a benchmark, we take the average of Comin (2004) and Chu (2010) and assume that R&D investment accounts for 0.5%, $g_Z = \iota \ln z = 0.005$. Thus, in a benchmark calibration, we set $g_A = 0.015$. Given $z = 1.05$, this assumption means that ι must be equal to $0.5/\ln(1.05) \approx 0.1205$ in the equilibrium.

Note that ι satisfies $\iota = B\Gamma(\underline{\varphi})l_{R,t}/N_t$ (see (18)). We use the Business Research and Development and Innovation Survey (BRDIS) published by the National Science Foundation to determine the target value of $l_{R,t}/N_t$. The BRDIS provides data on domestic R&D employment in the United States. To calculate the share of R&D employment, we sum the employment of scientists, engineers, their managers, and R&D technicians and technologists. We exclude R&D support staff such as clerical workers from R&D employment. According to the BRDIS, between 2008 and 2013, 6.8% of domestic employment in the sample accounted for R&D employment on average. Thus, as a benchmark, we assume that $l_{R,t}/N_t = 0.068$. These assumptions imply that $B\Gamma(\underline{\varphi}) = \iota/(l_{R,t}/N_t) \approx 1.5071$. Since the period between 2008 and 2013 includes the so-called “zero-interest rate policy,” we set $i = 0$ as a benchmark. Then, we choose the values of θ_R and B as well as the parameters concerning R&D firms' productivity to obtain $B\Gamma(\underline{\varphi}) = \iota/(l_{R,t}/N_t) \approx 1.5071$ for the homogeneous and heterogeneous cases, respectively.

Before proceeding to each case, we mention the following points. We also calibrate the parameters of our model, assuming that $g_Z = \iota \ln z = 0.002$ and $g_Z = \iota \ln z = 0.008$. The former corresponds to Comin's argument, while the latter corresponds to Chu's argument. We also present the calibration under $i = 0.05$.

The BRDIS includes only existing institutions and thus it does not include R&D investments conducted by start-up firms. In addition, the BRDIS does not include companies with fewer than five domestic employees. Then, our assumption, $l_{R,t}/N_t = 0.068$, may be inaccurate. Between 2010 and 2013, the average shares of employment of firms with 5–9 and 10–24 employees were only 0.7% and 1.8% of domestic employment in the samples, respectively. Thus, we conjecture that the employment of start-up firms and firms with fewer than five employees are small. Hence, including the R&D employment of these firms may not alter $l_{R,t}/N_t = 0.068$ significantly. The BRDIS also offers the amount of full-time-equivalent domestic R&D employment. Between 2008 and 2010, the average share of full-time-equivalent domestic R&D employment was 4.7%, if we exclude R&D support staff. Thus, the following discussion reports the results obtained under $l_{R,t}/N_t = 0.047$.

Homogeneous Productivity in the R&D Sector. When there is no heterogeneity among R&D firms, (26a) and (26b) characterize the equilibrium. The above procedure determines the value of $B\Gamma(\underline{\varphi})(= B\varphi)$. As long as the value of $B\varphi$ remains unchanged, the values of B and φ do not affect the equilibrium outcome. Thus, we need not calibrate B and φ. The parameter to be calibrated is θ_R. Given the values of z, ψ, σ, and ρ and the assumptions of g_A, ι, and $B\Gamma(\underline{\varphi})(= B\varphi)$, we use (26b) to determine the value of Q. After that, we use (26a) to determine the value of θ_R. Under the benchmark assumption, we get $Q \approx 1.0076$ and $\theta_R \approx 0.0841$. The value of $\underline{\theta_R}$ is 0.0154.

Heterogeneous Productivity in the R&D Sector. In the presence of heterogeneity, (24b) and (28) characterize the equilibrium. As in Liu and Wang (2014), we assume that φ follows a Pareto distribution, $F(\varphi) = 1 - (\varphi_{min}/\varphi)^a$, where $a > 1$ and $\varphi_{min} > 0$. Then, (24b) can be written as

$$\iota = \frac{\frac{\Theta_R}{(a-1)(B\underline{\varphi})^a} - \rho - (\sigma - 1)g_A}{(\sigma - 1)\ln z}, \tag{35}$$

where $\Theta_R \equiv \theta_R(B\varphi_{min})^a$. We rewrite (28) as

$$\iota = \frac{\frac{\psi-1}{1+i}B\underline{\varphi} - \rho - (\sigma - 1)g_A}{(\sigma - 1)\ln z + 1 + \frac{\psi-1}{1+i}\frac{B\underline{\varphi}}{B\Gamma(\underline{\varphi})}}, \tag{36}$$

where $B\Gamma(\underline{\varphi}) = \frac{a}{a-1}B\underline{\varphi}$. The last term in the denominator is modified. In (35) and (36), we regard $B\underline{\varphi}$ as an endogenous variable, instead of $\underline{\varphi}$. Given the values of z, ψ, σ, and ρ and the assumptions of g_A, ι, and $B\Gamma(\underline{\varphi})$, (36) determines the value of $B\underline{\varphi}$. Then, we use $B\Gamma(\underline{\varphi}) = \frac{a}{a-1}B\underline{\varphi}$ and the assumption of $B\Gamma(\underline{\varphi})$ to determine the value of a. We finally use (35) to determine the value of Θ_R. Note that as long as Θ_R remains unchanged, the values of θ_R, B, and φ_{min} do not affect the equilibrium outcome. Thus, we do not calibrate these three parameters. We use Θ_R as a measure of the severity of financial frictions. We denote the lower bound of Θ_R^H as $\underline{\Theta_R}^H \equiv \underline{\theta_R}^H(B\varphi_{min})^a$. Under the benchmark assumption, we get $B\underline{\varphi} \approx 0.8225$, $a \approx 2.2015$ and $\Theta_R \approx 0.0547$. The value of a satisfies (34). The value of $\underline{\Theta_R}^H$ is 0.0051.

5.2 Numerical Results

The four panels in Figure 1 plot the right-hand side of (31) evaluated at $i = 0$ against the measure of financial frictions (θ_R or Θ_R). The vertical lines in each panel show the calibrated value of θ_R or Θ_R. In all cases, under the calibrated value of θ_R or Θ_R, the Friedman rule is not optimal. The upper two panels use the benchmark calibration parameters, except for θ_R or Θ_R. Panel (a) shows that without heterogeneity in R&D firms' productivity, the Friedman rule is optimal if financial frictions are severe (θ_R is low), while the Friedman rule is not optimal if financial frictions are mild (θ_R is high). This corresponds to case (ii) in Proposition 3. Panel (b) shows that in the presence of heterogeneity in R&D firms' productivity, the opposite result holds: under mild financial frictions, the Friedman rule is optimal, while under severe financial frictions (Θ_R is small), the Friedman rule is not optimal, as discussed in Section 4.3.

[Figure 1]

Panel (b) further shows that financial frictions affect the between- and within-sector effects. With severe financial frictions, there is a positive between-sector effect, while there is a negative between-sector effect with mild financial frictions. This is because with severe (mild) financial frictions, the aggregate productivity of the R&D sector becomes low (high) and hence L_R becomes small (large), which means that a small (large) amount of labor should be allocated to the R&D sector. As financial frictions become milder (Θ_R increases), the within-sector effect increases. If financial frictions are severe, only a small amount of labor engages in R&D activity. Then, even if the aggregate productivity of the R&D sector improves, R&D intensity ι changes little. Thus, the within-sector effect becomes small. However, if financial frictions are mild, a large amount of labor engages in R&D activity, which means that even a slight improvement in the aggregate productivity of the R&D sector has a large impact on R&D intensity ι. Thus, the within-sector effect becomes large. The lower two panels use the calibrated parameters, assuming that $i = 0.05$. These two panels show that the results are not affected qualitatively.

The panels in Figure 2 plot U and the inflation rate against i, using the benchmark calibration except for i, θ_R and Θ_R. The left-hand vertical axis of each panel measures U, while the right-hand one measures the inflation rate. The upper five panels are for the homogeneous productivity case, while the lower five panels are for the heterogeneous productivity case. If we move from the left panels to the right ones, financial frictions become milder. The second panels from the left present the results under the calibrated values of θ_R and Θ_R. These two panels show that there is an optimal nominal interest rate and an optimal inflation rate in both the homogeneous and the heterogeneous R&D productivity cases. The purpose here is not to examine the exact value of the optimal nominal interest rate. Instead, we focus on the qualitative effects of financial frictions on the optimal nominal interest rate. If there is no heterogeneity in R&D firms' productivity, the optimal interest and inflation rates decrease as financial frictions become severer (θ_R decreases), as shown in Proposition 4. By contrast, with heterogeneity in R&D firms' productivity, the optimal interest and inflation rates increase as financial frictions become severer (Θ_R decreases). As financial frictions become severer, low-productivity R&D firms become operative, which results in lower aggregate R&D productivity. Then, L_R becomes smaller and hence labor employment in the R&D sector becomes inefficiently larger ($l_{R,t}/N_t > L_R$). An increase in the nominal interest rate mitigates the effects of severer financial frictions by making low-productivity R&D firms inoperative and reallocating labor from the R&D sector to the intermediate goods sector. Thus, the optimal nominal interest rate increases as Θ_R decreases. These panels also show that the welfare effects may be small quantitatively.

[Figure 2]

The four panels of Figure 3 show the right-hand side of (31) evaluated at $i = 0$ under different assumptions about $g_Z (= \iota \ln z)$ and $l_{R,t}/N_t$. The left two panels are for the homogeneous R&D productivity cases. Since we have $L_R < 0$ if $g_Z = 0.002$, which violates one of the assumptions in Proposition 3, we do not present the results for $g_Z = 0.002$. In all cases, with severe financial frictions (low θ_R), the Friedman rule is optimal, as shown in Proposition 3. If we assume $g_Z = 0.008$ or $l_{R,t}/N_t = 0.47$, the Friedman rule becomes optimal for all $\theta_R (> \underline{\theta_R})$, which corresponds to case (i) in Proposition 3. The right two panels in Figure 3 are for the heterogeneous R&D productivity cases. In all cases, with severe financial frictions (low Θ_R), deviating from the Friedman rule improves social welfare, as we show in Proposition 5.

[Figure 3]

Figure 3 shows that the range of θ_R and Θ_R under which the Friedman rule is optimal depends on the assumptions of $g_Z (= \iota \ln z)$ and $l_{R,t}/N_t$. Note that $B\Gamma(\underline{\varphi}) = \iota/(l_{R,t}/N_t) = (g_Z/\ln z)/(l_{R,t}/N_t)$. A large g_Z or a small $l_{R,t}/N_t$ implies high aggregate productivity in the R&D sector. Figure 3 suggests that if the aggregate productivity of the R&D sector is high, the Friedman rule tends to be optimal for a wider range of financial frictions.

6 Financial Frictions and Cash on Intermediate Goods

This section constructs a model in which intermediate goods firms face a CIA constraint and have an incentive to default. To save the notations, we use the same notations used in the benchmark model, as far as we can.

We focus on the intermediate goods sector. Between time t and $t + dt$, intermediate goods firm j behaves as follows. First, it sets $p_{j,t}$, which determines the amount of labor input, $l_{x,j,t}$, through (2) and (3). At the same time, firms determine the amount of real money, $m_{x,j,t}$, that they borrow from households. We assume that firms use money to pay the proportion $\beta \in [0, 1)$ of labor wages. Thus, the CIA constraint is given by $\beta w_t l_{x,\varphi,t} \cdot dt \leq m_{x,\varphi,t} \cdot dt$. The assumption $\beta < 1$ means that the CIA constraint in the R&D sector is tighter than that in the intermediate goods sector, reflecting the fact that firms rely on cash extensively to finance R&D. Since intermediate firms cannot make upfront payments to workers and cannot repay $m_{x,j,t} \cdot dt$ to households until they earn revenue, workers and households are effectively providing credit to intermediate goods firms. Second, firm j earns the revenue of $p_{j,t} x_{j,t} = Y_t$, where the equality holds because of (2). Finally, firm j decides whether to repay the remaining labor wages, $(1 - \beta) w_t l_{x,j,t}$, and money, $(1 + i_t) m_{x,j,t}$. If firm j defaults, it is caught with probability $\theta_x \cdot dt$ ($\theta_x > 0$). Its revenue is seized and the firm is perpetually excluded from future access to credit.

We derive an incentive constraint for firm j. If firm j defaults, its value in terms of the final good becomes $q^N_{j,t}$. Otherwise, the value is $q^D_{j,t}$. We define $q_{j,t} = \max\{q^N_{j,t},\ q^N_{j,t}\}$. Then, we have

$$
\begin{aligned}
q^N_{j,t} &= \left[p_{j,t} x_{j,t} + (m_{x,j,t} - \beta w_t l_{x,j,t}) - (1 - \beta) w_t l_{x,j,t} - (1 + i_t) m_{x,j,t} \right] dt + \frac{1 - \iota_t \cdot dt}{1 + r_t \cdot dt} q_{j,t+dt}, \\
q^D_{j,t} &= \left[p_{j,t} x_{j,t} + (m_{x,j,t} - \beta w_t l_{x,j,t}) \right] dt + \frac{1 - \iota_t \cdot dt - \theta_x \cdot dt}{1 + r_t \cdot dt} q_{j,t+dt}.
\end{aligned}
$$

Here, we take into account the fact that innovation happens at the rate of $\iota_t \cdot dt$. If $q^N_{j,t} \geq q^D_{j,t}$, firm j has no incentive to default and we have $q_{j,t} = q^N_{j,t}$. From $q^N_{j,t} \geq q^D_{j,t}$, we can derive the following financial constraint:

$$
(1 - \beta) w_t l_{x,j,t} + (1 + i_t) m_{x,j,t} \leq \theta_x q_{j,t+dt}. \tag{37}
$$

We show that (37) never binds in the equilibrium. Firm j chooses $p_{j,t}$ and $m_{x,j,t}$ to maximize $q_{j,t}$ subject to (37). If we use the demand function, (2), $q_{j,t}(= q^N_{j,t})$ is given by

$$q_{j,t} = \left[Y_t + (m_{x,j,t} - \beta w_t l_{x,j,t}) - (1-\beta) w_t l_{x,j,t} - (1+i_t) m_{x,j,t}\right] dt + \frac{1 - \iota_t \cdot dt}{1 + r_t \cdot dt} q_{j,t+dt}.$$

Suppose that (37) holds with equality. The above equation shows that since the demand function, (2), has unit price elasticity, firm j's choice of $p_{j,t}$ and $m_{x,j,t}$ has no influence on its revenue $p_{j,t}x_{j,t} = Y_t$. Thus, firm j can increase $q_{j,t}$ by reducing $l_{x,j,t}$ and $m_{x,j,t}$. This makes the financial constraint loose and unbinding. Hence, (37) never binds in the equilibrium.

In the presence of the CIA constraint, the total cost of producing intermediate good j is $(1+\beta i_t) w_t l_{x,j,t}$. The marginal cost of producing intermediate good j is given by $(1+\beta i_t)w_t/z^{s_{j,t}}$. Thus, we have $p_{j,t} = \psi(1+\beta i_t)w_t/z^{s_{j,t}}$, $x_{j,t} = z^{s_{j,t}} Y_t/[\psi(1+\beta i_t)w_t]$, and $l_{x,j,t} = Y_t/[\psi(1+\beta i_t)w_t]$. The operating profit, $\pi_{x,j,t}$, is given by (4b), as before. Since firms earn the same levels of operating profits, $q_{j,t}$ becomes independent of j, $q_{j,t} = q_t$, where q_t satisfies (5), as before. The second equation of (6) still holds. However, the first one is modified as $w_t = A_t Z_t/[(1+\beta i_t)\psi]$.

In the benchmark model, the steady state equilibrium is characterized by three equations, namely (24a), (24b), and (24c). Of these three equations, (24b) and (24c) are not affected by the extension in this section. (24a) is modified as

$$\underline{\varphi} = \frac{1+i}{B\psi(1+\beta i)} Q.$$

Given Q, as long as $\beta \in [0,1)$, β does not affect the qualitative effect of i on $\underline{\varphi}$. Thus, introducing a CIA constraint in the intermediate goods sector does not alter Propositions 1–5 if $\beta \in [0,1)$. Of course, this extension quantitatively affects the level of the optimal nominal interest rate. However, the qualitative characteristics of the optimal nominal interest rate are not affected.

7 A Variety-Expansion Model

This section observes that under some conditions, the benchmark quality-ladder model becomes identical to a standard model of expanding product variety and hence all of the results thus far hold. As in Section 6, we use the same notations used in the benchmark model, as far as we can. For simplicity, we do not consider the financial and CIA constraints in the intermediate goods sector. This can be justified by the discussion in Section 6. Since the variety-expansion model has been well studied, we describe the model briefly.

The final good is produced by

$$Y_t = A_t \cdot \Omega_t^{\gamma - \frac{1-\xi}{\xi}} \left[\int_0^{\Omega_t} x_{j,t}{}^{\xi}\right]^{\frac{1}{\xi}},$$

where $\Omega_t > 0$ is the variety of intermediate goods, $\xi \in (0,1)$ is a parameter, and $\gamma > 0$ represents the gains from the increased division of labor. This formulation follows Ethier (1982). Product variety Ω_t increases through R&D activities. Demand for good j is given by

$$x_{j,t} = \frac{Y_t p_{j,t}^{-\frac{1}{1-\xi}}}{\int_0^{\Omega_t} p_{j,t}^{-\frac{\xi}{1-\xi}} dj}.$$

A unit of labor produces a unit of intermediate good j. Firm j maximizes its operating profits, $\pi_{x,j,t} = (p_{j,t} - w_t)x_{j,t}$, subject to the above demand function, which yields

$$p_{j,t} = \frac{w_t}{\xi}(\equiv p_t), \quad x_{j,t} = \frac{Y_t}{\Omega_t p_t}(\equiv x_t = l_{x,t}), \quad \text{and } \pi_{x,j,t} = (1-\xi)\frac{Y_t}{\Omega_t}(\equiv \pi_{x,t}).$$

The value of intermediate goods firm, q_t, satisfies $r_t q_t = \pi_{x,t} + \dot{q}_t$. The two equations in (6) are modified as

$$w_t = \xi A_t \Omega_t^{\gamma} \quad \text{and} \quad Y_t = A_t \Omega_t^{\gamma} \cdot \Omega_t l_{x,t},$$

where $\Omega_t l_{x,t}$ is the total labor allocated to intermediate goods production.

If an R&D firm with productivity φ employs $l_{R,\varphi,t}$ units of labor at time t, new intermediate goods are invented according to

$$\dot{\Omega}_{\varphi,t} = \Omega_t \cdot B\varphi \frac{l_{R,\varphi,t}}{N_t},$$

where Ω_t represents knowledge spillover. The other settings for the R&D sector are the same as in the benchmark model. If we redefine $\underline{\varphi}_t \equiv (1+i_t)w_t N_t/(Bq_t\Omega_t)$, we can derive the same equations as (13), (15), and (16). The aggregate growth rate of Ω_t is given by $g_{\Omega,t} = \int(\dot{\Omega}_{\varphi,t}/\Omega_t)dF(\varphi) = B\int(\varphi l_{R,\varphi,t}/N_t)dF(\varphi)$. If we replace ι_t with $g_{\Omega,t}$, (18) still holds.

The preference of the representative household is given by (19). The second equation of (20) is modified as $\dot{\eta}_t = \omega - n\eta_t$. Utility maximization yields the Euler equation, (22). The monetary authority behaves in the same way as in the benchmark model.

Since the labor market equilibrium requires $N_t = l_{R,t} + \Omega_t l_{x,t}$, (23) is modified as

$$\frac{\Omega_t l_{x,t}}{N_t} = 1 - \frac{g_{\Omega,t}}{B\Gamma(\underline{\varphi})}.$$

In the above equation, we use (18), of which ι_t is replaced with $g_{\Omega,t}$.

In the steady state, $g_{\Omega,t}$, φ, $l_{R,t}/N_t$, and $\Omega_t l_{x,t}/N_t$ become constant over time. To study the steady state, we define $Q_t \equiv \overline{A}_t\Omega_t^{\gamma}N_t/(q_t\Omega_t)$ and $V_t \equiv A_t\Omega_t^{\gamma}N_t/v_t$. In the steady state, the growth rate of per capita output is given by $g = g_A + \gamma g_{\Omega,t}$. The steady state is characterized by

$$\begin{aligned}
\underline{\varphi} &= \frac{\xi(1+i)Q}{B}, \\
g_{\Omega,t} &= \frac{\theta_R \int_{\underline{\varphi}}^{\infty}\left(\frac{\varphi}{\underline{\varphi}} - 1\right)dF(\varphi) - \rho - (\sigma-1)g_A}{(\sigma-1)\gamma}, \\
g_{\Omega,t} &= \frac{(1-\xi)Q - \rho - (\sigma-1)g_A}{(\sigma-1)\gamma + 1 + (1-\xi)\frac{Q}{B\Gamma(\underline{\varphi})}}.
\end{aligned}$$

If we set $\gamma = \ln z$ and $\xi = 1/\psi$ and replace $g_{\Omega,t}$ with ι, these three equations become identical to (24a), (24b), and (24c), respectively. Thus, all the results of the benchmark model hold in the variety-expansion model described in this section.

8 Conclusion

The empirical facts show that (a) R&D investments are exposed to financial constraints, (b) heterogeneity in R&D productivity matters in the presence of financial constraints, and (c) R&D firms rely on cash to finance R&D expenditure. These facts suggest that monetary policy could have important effects on R&D activities. Thus, we construct an R&D-based endogenous growth model that reflects these three facts.

If there are severe financial constraints, the optimality of the Friedman rule depends crucially on the presence of heterogeneous R&D productivity. Without heterogeneity, the Friedman rule is optimal. By contrast, with heterogeneity, social welfare is improved by deviating from the Friedman rule under a plausible condition. The numerical analysis shows that there is a unique optimal nominal interest rate. Without heterogeneity, as financial constraints become severer, the optimal nominal interest rate decreases. With heterogeneity, the opposite result is obtained. Our results indicate that ignoring financial constraints on R&D investments and heterogeneous R&D productivity might result in inappropriate recommendations for monetary policy.

Appendix

A Derivation of (24a), (24b), and (24c)

From the final goods market equilibrium, $c_t N_t = Y_t$, we have

$$r_t - \rho - n = \sigma\left(g_A + \iota_t(\ln z) + \frac{1}{l_{x,t}/N_t}\frac{\partial(l_{x,t}/N_t)}{\partial t}\right), \tag{A.1}$$

where we use the second equation of (6), (22), and $g_Z \equiv \dot{Z}_t/Z_t = \iota_t \ln z$. We use (4b), (5), (15), the second equation of (6), and the definition of Q_t and V_t to derive the following two equations:

$$\frac{\dot{V}_t}{V_t} = g_A + \iota_t \ln z + n - \left\{r_t - \theta_R \int_{\underline{\varphi}_t}^{\infty}\left(\frac{\varphi}{\underline{\varphi}_t} - 1\right) dF(\varphi)\right\}, \tag{A.2}$$

$$\frac{\dot{Q}_t}{Q_t} = g_A + \iota_t \ln z + n - \left(r_t + \iota_t - \frac{\psi - 1}{\psi} Q_t \frac{l_{x,t}}{N_t}\right). \tag{A.3}$$

We set $\partial(l_{x,t}/N_t)/\partial t = \dot{V}_t = \dot{Q}_t = 0$ in (A.1), (A.2), and (A.3). We eliminate r_t from (A.1) and (A.2) and then rearrange the resulting equation to obtain (24b). Similarly, we obtain (24c) from (A.1) and (A.3). Substituting the first equation of (6) into the definition of $\underline{\varphi}_t$ yields (24a). Setting $\dot{Q}_t = 0$ in (A.3) and using $r = \rho + n + \sigma[g_A + \iota(\ln z)]$ yield (25).

B Proof of Proposition 1

$Q_a(\theta_R, i)$ is an increasing function of θ_R that satisfies $Q_a(0, i) = 0$. Then, if $\lim_{\theta_R \to +\infty} Q_a(\theta_R, i) > Q_b$, there exist a unique $\underline{\theta_R}$ that satisfies $Q_a(\underline{\theta_R}, i) = Q_b$ and $Q_a(\theta_R, i) > Q_b$ for $\theta_R > \underline{\theta_R}$.

Suppose that $\theta_R < +\infty$. Because of $\sigma > 1$, (26a) shows that ι decreases with Q and takes a positive value for $Q < Q_a(\theta_R)$. We differentiate (26b) with respect to Q:

$$\frac{d\iota}{dQ} = \frac{\frac{\psi-1}{\psi}\frac{l_{x,t}}{N_t}}{(\sigma-1)\ln z + 1 + \frac{\psi-1}{\psi}\frac{Q}{B\varphi}} > 0,$$

where we use (23). Then, (26b) is an increasing function of Q that takes a positive value for $Q > Q_b$. As shown in Figure 4 (a), if $\theta_R > \underline{\theta}_R$ ($Q_a(\theta_R, i) > Q_b$), the graphs of (26a) and (26b) have a unique intersection such that $\iota > 0$. Next, suppose that $\theta_R = +\infty$. To draw the graph of (26a), we divide both sides of (26a) by θ_R and take the limit of $\theta_R \to \infty$, which results in $Q = \frac{\psi B\varphi}{1+i}(= \lim_{\theta_R\to+\infty} Q_a(\theta_R, i))$. Thus, when $\theta_R = +\infty$, the graph of (26a) becomes vertical. Because of $\lim_{\theta_R\to+\infty} Q_a(\theta_R, i) > Q_b$, the graphs of (26a) and (26b) have a unique intersection such that $\iota > 0$ (see Figure 4 (b)).

[Figure 4]

Irrespective of whether $\theta_R < +\infty$ or $\theta_R = +\infty$, an increase in i shifts the graph of (26a) leftward. When $\theta_R < +\infty$, an increase in θ_R shifts the graph of (26a) rightward. As $\theta_R \to \underline{\theta_R}$, the intersection of (26a) and (26b) approaches $(Q, \iota) = (Q_b, 0)$. Then, the effects of i and θ_R on ι in (27a) and (27b) are obtained. Because of $\Gamma(\underline{\varphi}) = \phi$, we have (27c).

C Proof of Proposition 2

From (29), we have $\underline{\theta_R}^H = [\rho + (\sigma-1)g_A]/\int_{\underline{\varphi}_b}^{\infty}\left(\frac{\varphi}{\underline{\varphi}_b} - 1\right)dF(\varphi)$. Since $\underline{\varphi}_a(\theta_R)$ increases with θ_R, we have $\underline{\varphi}_a(\theta_R) > \underline{\varphi}_b$ for $\theta_R > \underline{\theta_R}^H$. Since $\underline{\varphi}_b$ depends on i, $\underline{\theta_R}^H$ also depends on i.

Suppose that $\theta_R < +\infty$. We differentiate (24b) with respect to $\underline{\varphi}$:

$$\text{sign}\left\{\frac{d\iota}{d\underline{\varphi}}\right\} = -\theta_R\int_{\underline{\varphi}}^{\infty}\frac{\varphi}{\underline{\varphi}^2}dF(\varphi) < 0.$$

Then, (24b) is a decreasing function that takes a positive value for $\underline{\varphi} < \underline{\varphi}_a(\theta_R)$. We next differentiate (28) with respect to $\underline{\varphi}$:

$$\text{sign}\left\{\frac{d\iota}{d\underline{\varphi}}\right\} = \frac{l_{x,t}}{N_t} + \frac{\underline{\varphi}\Gamma'(\underline{\varphi})\iota}{B\{\Gamma(\underline{\varphi})\}^2} > 0,$$

where we use (23). Then, (28) is an increasing function that takes a positive value for $\underline{\varphi} > \underline{\varphi}_b(\theta_R)$. As shown in Figure 5 (a), if $\underline{\varphi}_a(\theta_R) > \underline{\varphi}_b$ ($\theta_R > \underline{\theta_R}^H$), the graphs of (24b) and (28) have a unique intersection with $\iota > 0$. Next, suppose that $\theta_R = \infty$ and $\varphi_{max} < +\infty$. To draw the graph of (24b), we divide both sides of (24b) by θ_R and take the limit of $\theta_R \to +\infty$, which results in $\underline{\varphi} = \varphi_{max}(= \lim_{\theta_R\to+\infty}\underline{\varphi}_a(\theta_R))$. The graph of (24b) becomes vertical. Because of $\varphi_{max} > \varphi_b$, the graphs of (24b) and (28) have a unique intersection with $\iota > 0$ (see Figure 5 (b)).

[Figure 5]

Irrespective of whether $\theta_R < +\infty$ or $\theta_R = +\infty$, an increase in i shifts the graph of (28) rightward. When $\theta_R < +\infty$, an increase in θ_R shifts the graph of (24b) rightward. Thus, we obtain (30a) and (30b). When θ_R approaches $\underline{\theta_R}^H$, the intersection of the two graphs approaches $(\underline{\varphi}, \iota) = (\underline{\varphi}_b, 0)$, which results in the third equation of (30b). Note that we have $\Gamma'(\underline{\varphi}) > 0$ if $\theta_R < +\infty$, while we have $\Gamma(\underline{\varphi}) = \varphi_{max}$ if $\theta_R = +\infty$. Thus, we have (30c).

References

Acemoglu, D. and Akcigit, U. (2012) “Intellectual property rights policy, competition and innovation,” *Journal of the European Economic Association* 10, 1-42.

Arrow, K. (1962) “Economic Welfare and the Allocation of Resources for Invention,” (pp. 609—625), in R. R. Nelson (Ed.), The Rate and Direction of Inventive Activity: Economic and Social Factors (Princeton, NJ: Princeton University Press).

Bates, T. W., Kahle, K. M., and Stulz, R. M. (2009) “Why do US firms hold so much more cash than they used to?” *Journal of Finance* 64, 1985-2021.

Bond, S., Harhoff, D., and Van Reenen, J. (2006) “Investment, R&D and financial constraints in Britain and Germany,” *Annales d’Economie et de Statistique* 79-80, 1—28.

Berentsen, A., Breu, M. R., and Shi, S. (2012) “Liquidity, innovation and growth,” *Journal of Monetary Economics* 59, 721-737.

Brown, J. R., Fazzari, S. M., and Petersen, B. C. (2009) “Financing innovation and growth: Cash flow, external equity, and the 1990s R&D boom,” *The Journal of Finance* 64, 151-185.

Brown, J. R., Martinsson, G., and Petersen, B. C. (2012) “Do financing constraints matter for R&D?” *European Economic Review* 56, 1512-1529.

Brown, J. R. and Petersen, B. C. (2009) “Why has the investment-cash flow sensitivity declined so sharply? Rising R&D and equity market developments,” *Journal of Banking & Finance* 33, 971-984.

Brown, J. R. and Petersen, B. C. (2011) “Cash holdings and R&D smoothing,” *Journal of Corporate Finance* 17, 694-709.

Chari, V. V. and Kehoe, P. J. (1999) Optimal Fiscal and Monetary Policy. Handbook of Macroeconomics, vol. 1C. Elsevier.

Chu, A. (2010) “Effects of patent length on R&D: A quantitative analysis,” *Journal of Economics* 99, 117-140

Chu, A. and Cozzi, G. (2014) “R&D and economic growth in a cash-in-advance economy,” *International Economic Review* 55, 507-524.

Chu, A. and Lai, C-H. (2013) “Money and the welfare cost of inflation in an R&D growth model,” *Journal of Money, Credit and Banking* 45, 233-249.

Comin, D. (2004) “R&D: A small contribution to productivity growth,” *Journal of Economic Growth* 9, 391-421.

Ethier, W. J. (1982) “National and international returns to scale in the modern theory of international trade,” *American Economic Review* 72, 389-405.

Evers, M., Niemann, S., and Schiffbauer, M. (2009) “Inflation, liquidity risk, and long-run TFP-growth,” Economic and Social Research Institute Paper DYNREG 48.

Falato, A. and Sim, J. (2014) "Why do innovative firms hold so much cash? Evidence from changes in state R&D tax credits," Staff working papers in the Finance and Economics Discussion Series (FEDS) No. 2014-72.

Funk, P. and Kromen, B. (2010) "Inflation and innovation-driven growth," *BE Journal of Macroeconomics* 10.1, 50.

Goh, A-T. and J. Olivier. (2002) "Optimal patent protection in a two-sector economy," *International Economic Review* 43, 1191-1214.

Gorodnichenko, Y. and Schnitzer, M. (2013) "Financial constraints and innovation: Why poor countries don't catch up." *Journal of the European Economic Association* 11, 1115-1152.

Grossman, G. M. and Helpman, E. (1991) Innovation and growth in the global economy. MIT Press.

Hajivassiliou, V. and Savignac, F. (2008) "Financing constraints and a firm's decision and ability to innovate: Establishing direct and reverse effects," Banque de France, Notes d'Études de Recherche NER-R no. 202.

Himmelberg, C. P. and Petersen, B. C. (1994) "R&D and internal finance: A panel study of small firms in high-tech industries," *Review of Economics and Statistics* 76, 38-51.

Hottenrott, H. and Peters, B. (2012) "Innovative capability and financing constraints for innovation: More money, more innovation?" *Review of Economics and Statistics* 94, 1126-1142.

Jones, C. and Williams, J. (2000) "Too much of a good thing? The economics of investment in R&D," *Journal of Economic Growth* 5, 65-85.

Laincz, C. A. and Peretto, P. F. (2006) "Scale effects in endogenous growth theory: An error of aggregation not specification," *Journal of Economic Growth* 11, 263-288.

Liu, Z. and Wang, P. (2014) "Credit constraints and self-fulfilling business cycles," *America Economic Journal: Macroeconomics* 6, 32-69.

Marquis, M. H. and Reffett, K. L. (1994) "New technology spillovers into the payment system," *Economic Journal* 104, 1123-1138.

Moll, B. (2014) "Productivity losses from financial frictions: Can self-financing undo capital misallocation?" *American Economic Review* 104, 3186-3221.

Mulkay, B., Hall, B., and Mairesse, J. (2001) "Firm level investment and R&D in France and the United States: A comparison," NBER Working Paper 8038.

Mulligan, C. B. and Sala-i-Martin, X. X. (1997) "The optimum quantity of money: Theory and evidence," *Journal of Money, Credit & Banking* 29, 687-716.

Oikawa, K. and Ueda, K. (2015) "The optimal inflation rate under Schumpeterian growth." mimeo.

Pinkowitz, L., Stulz, R. M., and Williamson, R. (2003) "Do firms in countries with poor protection of investor rights hold more cash?" National Bureau of Economic Research. No. w10188.

Ramírez, A. and Tadesse, S. (2009) “Corporate cash holdings, uncertainty avoidance, and the multinationality of firms,” *International Business Review* 18, 387-403.

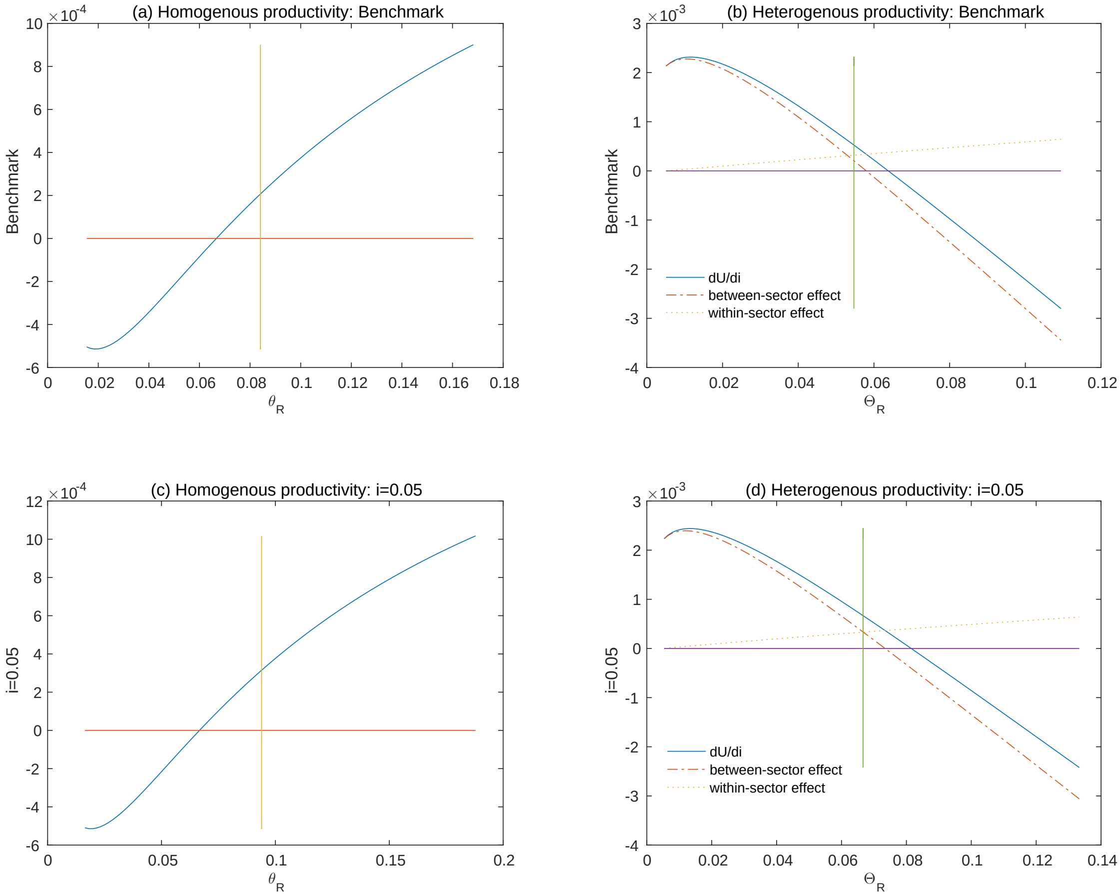

Figure 1: Effects of Financial Frictions

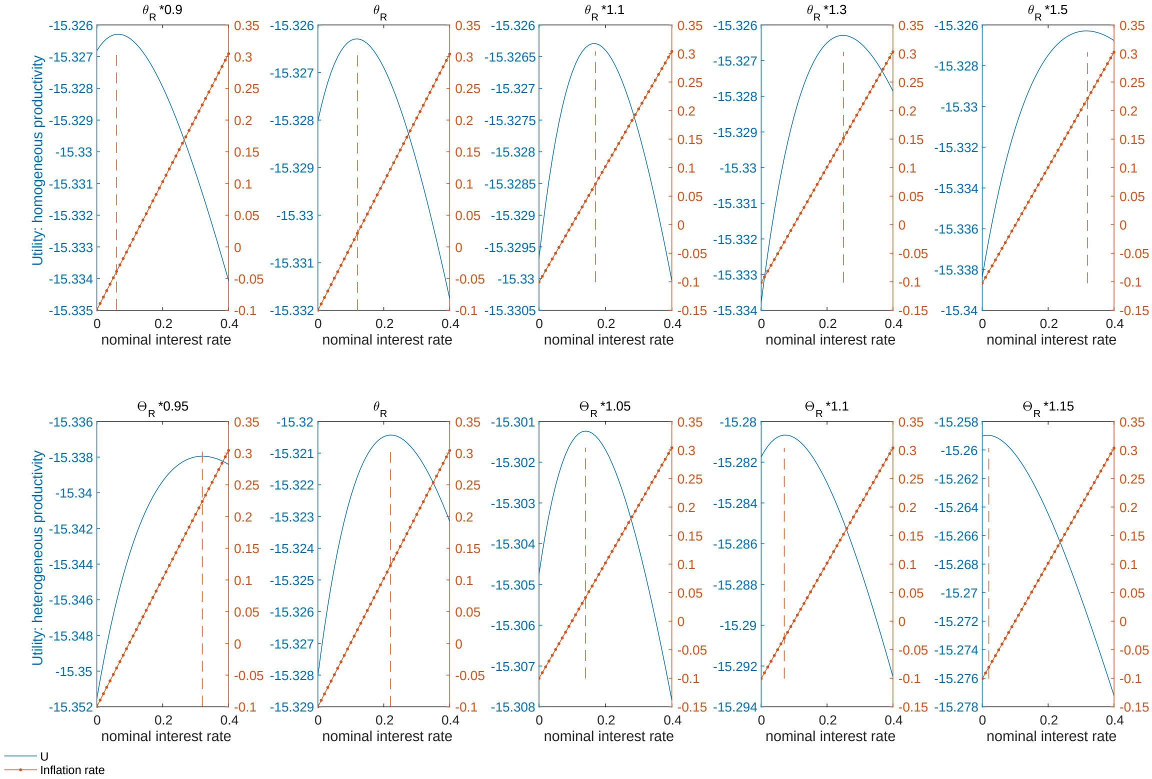

Figure 2: Effects of the Nominal Interest Rate

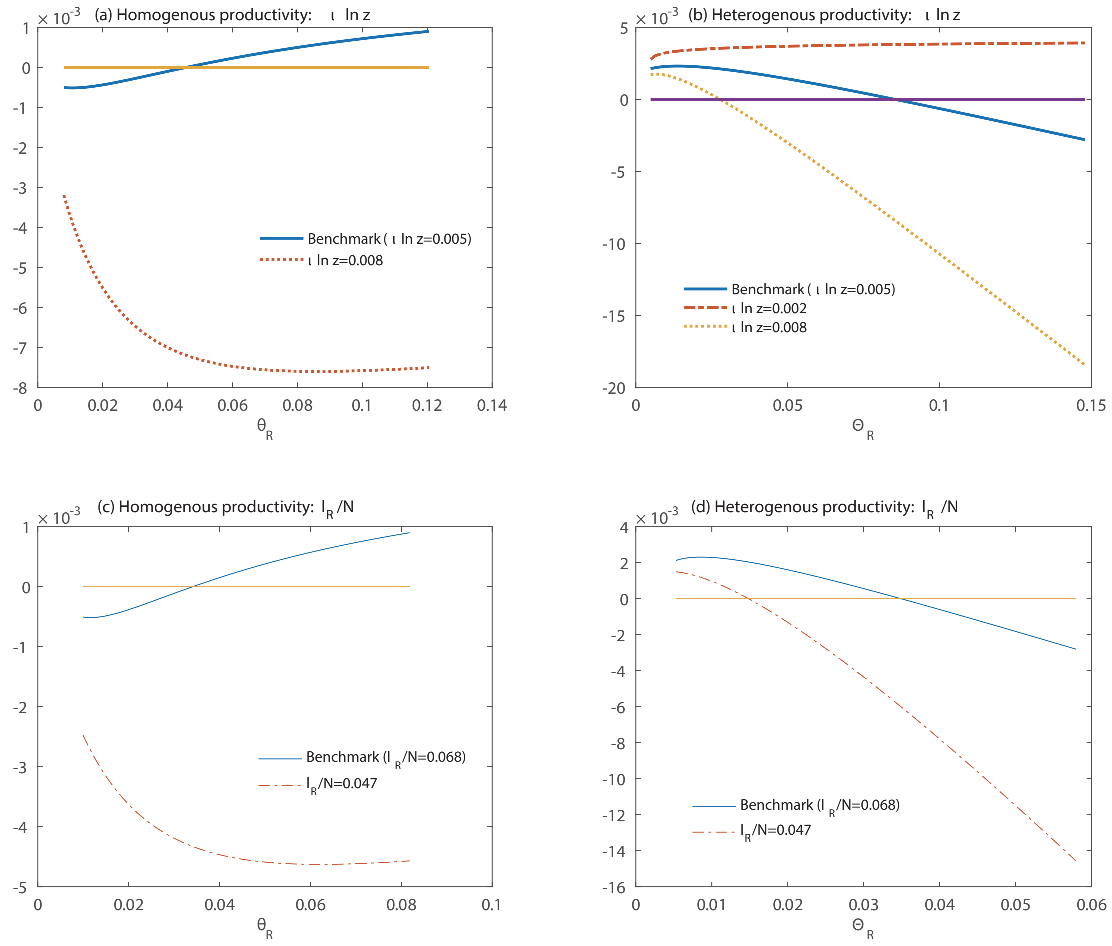

Figure 3: Changes in $\iota \ln z$ and $l_{R,t}/N_t$

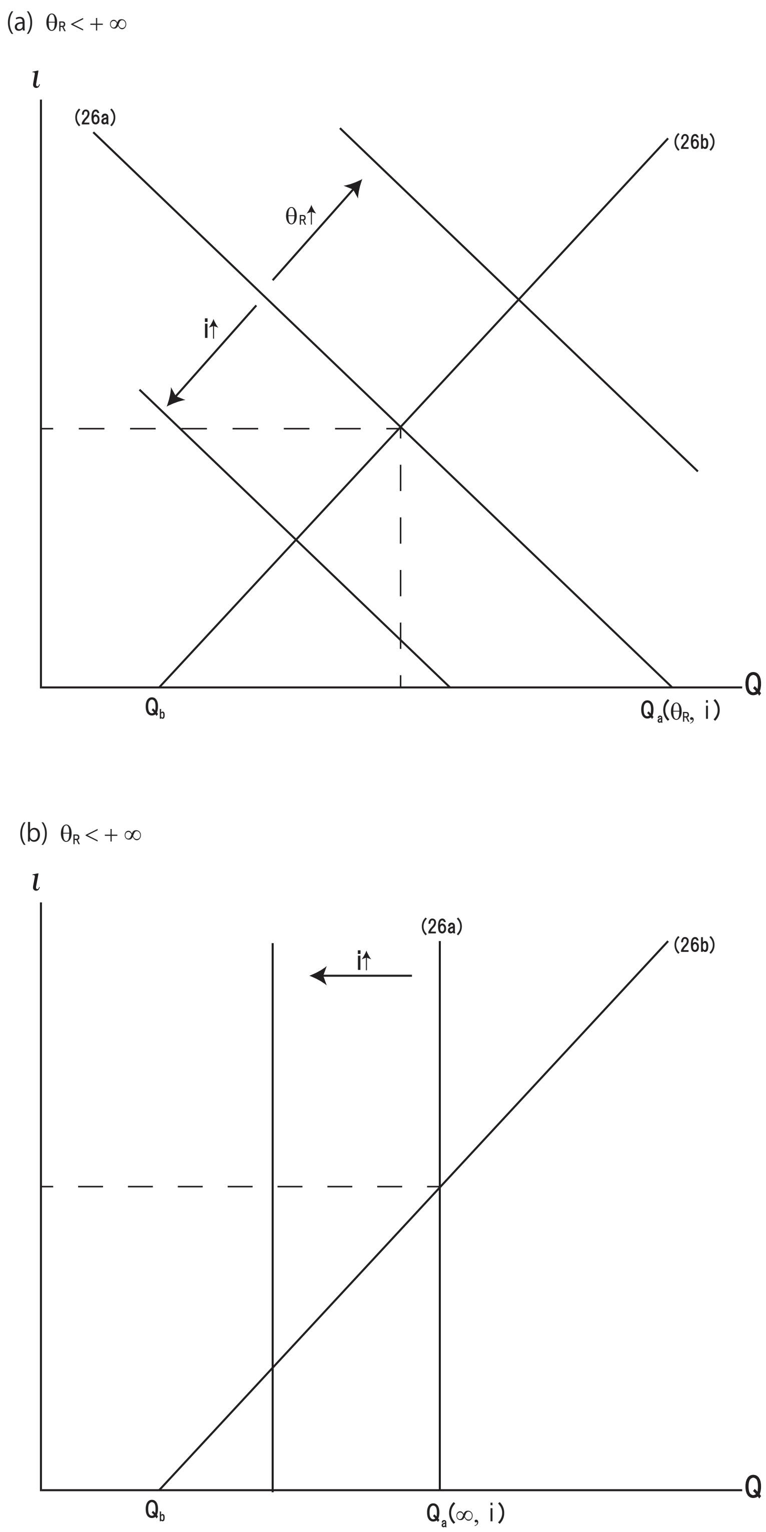

Figure 4: Steady State Equilibrium: Homogeneous Case

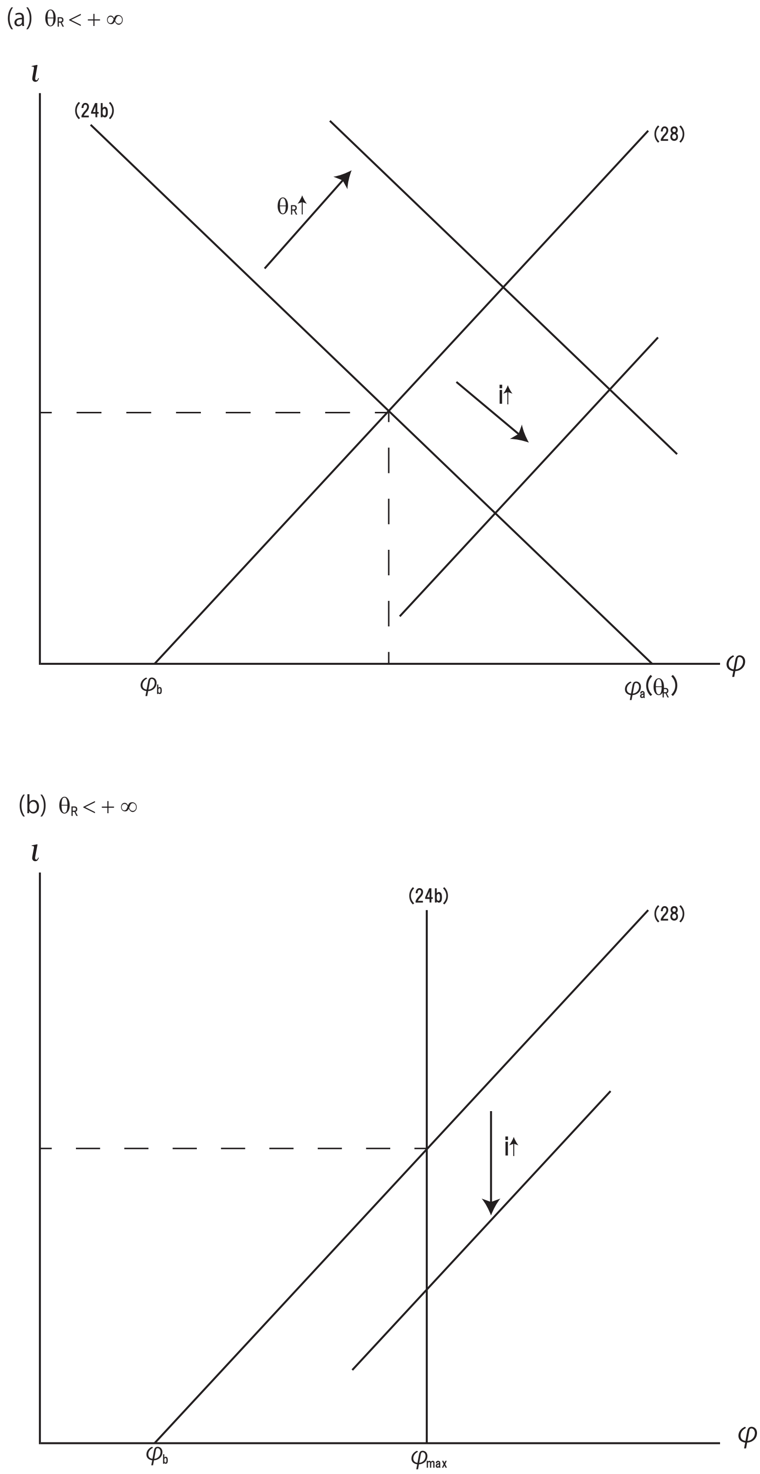

Figure 5: Steady State Equilibrium: Heterogeneous Case