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The pivotal mechanism versus the voluntary contribution mechanism: An experimental comparison*

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Abstract

We conduct an experimental comparison of two well-known mechanisms for undertaking a binary public project: the pivotal mechanism and the voluntary contribution mechanism. We observe that the voluntary contribution mechanism works better than the pivotal mechanism from the perspectives of Pareto efficiency and individual rationality, whereas we observe that no significant difference between the two mechanisms in terms of decision efficiency under complete information in which each subject knows the other subjects' payoffs. On the other hand, under incomplete information in which each subject knows their own payoff but not the other subjects' payoffs, there is no significant difference between the two mechanisms from the viewpoint of Pareto efficiency. The voluntary contribution mechanism performs better (resp. worse) than the pivotal mechanism considering individual rationality (resp. decision efficiency) under incomplete information.

Keywords: laboratory experiment; pivotal mechanism; voluntary contribution mechanism

JEL codes: C92; D71; D78; H41

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1 Introduction

We consider an economy with a public good in which agents are facing a decision of whether to undertake a project that produces a fixed level of the public good. The public project should be undertaken if the sum of agents' true values of the project is larger than or equal to the cost of undertaking the project, but not otherwise. However, this condition of decision efficiency is difficult to satisfy since each agent's true value of the public project is the agent's private information, and the agent may have an incentive to "free-ride" payments from other agents and undertake the project while lying about their own values.

There are two well-known mechanisms to solve this free-rider problem theoretically: the pivotal mechanism and the voluntary contribution (or provision point) mechanism. In the pivotal mechanism due to Clark (1977), reporting their true value of the project is a dominant strategy for each agent. Therefore, this mechanism satisfies the decision efficiency condition even under incomplete information in which each agent knows their own payoff but not the other agents' payoffs. However, the rule of the mechanism is complicated and dominant strategy equilibrium allocations of the mechanism may be neither Pareto efficient nor individually rational.

On the other hand, the rule of the voluntary contribution mechanism is simple, and the mechanism implements the core when the equilibrium concept is undominated perfect equilibrium, which is a refinement of Nash equilibrium (Bagnoli and Lipman, 1989). Then, all equilibrium allocations of the mechanism are Pareto efficient and individually rational. The mechanism also satisfies decision efficiency. However, this equilibrium concept requires complete information among agents: each agent knows the payoffs of all other agents. This suggests that the mechanism works well only under complete information.

To summarize, we cannot conclude theoretically which of these two mechanisms is better because each has advantages and disadvantages in different theoretical and normative aspects. Therefore, it would be worth investigating the performance of these mechanisms through empirical research such as economic experiments. In fact, several papers have examined how one of the two mechanisms works in laboratory experiments.[1] However, each of these experimental studies focused either on the pivotal mechanism or on the voluntary contribution mechanism. To our best knowledge, no paper experimentally compares the two mechanisms in the same economic

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[1] See Chen (2008) and Chen and Ledyard (2010) for excellent surveys on these experimental results.

environment with a public project.

The purpose of this paper is to investigate which mechanism, the pivotal mechanism or the voluntary contribution mechanism, should be used in the provision of a binary public project by conducting an experimental comparison. In designing an experiment to compare the two mechanisms, we pay close attention to the following two points. First, the comparison should be made under the same economic environmental conditions on the number of subjects, the true values that subjects receive from the project, the cost of undertaking the project, and the informational condition. For each subject's information on payoffs of other subjects' payoffs, we prepare the two different treatment conditions: the complete information treatment condition under which each subject knows the true values of all subjects, which is required in the concept of Nash equilibrium, and the incomplete information treatment condition under which each subject knows their own true value, but not the values to others, which is implicitly assumed in the concept of dominant strategy equilibrium. We compare the two mechanisms under complete information and under incomplete information. The theoretical predictions suggest that the voluntary contribution mechanism would perform better (resp. worse) than the pivotal mechanism under complete (resp. incomplete) information. We test whether these predictions are correct in a laboratory experiment.

The second point we are concerned with in comparing the two mechanisms is to hold the complexity constant. Clearly, the rule of the pivotal mechanism is more complicated than that of the voluntary contribution mechanism. Attiyeh et al. (2000) and Kawagoe and Mori (2001) argued that subjects suffered from confusion due to the complexity of the pivotal mechanism and the non-transparency of the dominant strategy. To eliminate such confusion, following Cason et al. (2006), we focused on the two-agent case and simply presented to each subject a payoff table describing the relationship between outcomes and choices by the subject and the others without explaining the rule of the pivotal mechanism. Similarly, for the voluntary contribution mechanism experiment, we used payoff tables only and gave no explanation on the rule of the mechanism. The use of payoff tables allows for a comparison of the pivotal mechanism with the voluntary contribution mechanism holding the degree of transparency constant.

We compare the two mechanisms from four different normative standards: Pareto efficiency, individual rationality, decision efficiency, and surplus indices. Here, the Pareto efficiency index is the ratio of Pareto efficient outcomes. The individual rationality index is the ratio of individually rational outcomes in which each subject receives a payoff at least as large as their own before participating in the mechanism, that is, the payoff the subject gets from the initial endowment of a private good without any public project. The decision efficiency index is the ratio of decision-efficient outcomes in which the public project is undertaken because the sum of the subjects' true values of the project exceeds the cost of undertaking the project in our experimental setting. The surplus index is a measure of the gains subjects obtain by participating in a mechanism, which is a percentage of the highest possible payoff that could be achieved in the mechanism.

We observed in the complete information treatments that the Pareto efficiency, individual rationality, and surplus indices for the voluntary contribution mechanism were significantly higher than those for the pivotal mechanism. Additionally, there was no significant difference in the decision efficiency index between the two mechanisms. These results suggest that the voluntary contribution mechanism would work better than the pivotal mechanism under complete information.

Under incomplete information, we found that neither the Pareto efficiency index nor the surplus index for the voluntary contribution mechanism were significantly different from those for the pivotal mechanism. The individual rationality index for the voluntary contribution mechanism was significantly higher than the individual rationality index for the pivotal mechanism, whereas the decision efficiency index for the voluntary contribution mechanism was significantly lower than the decision efficiency index for the pivotal mechanism. Therefore, the voluntary contribution (resp. pivotal) mechanism would be superior to the pivotal (resp. voluntary contribution) mechanism if individual rationality is more (resp. less) important than decision efficiency, although we cannot conclude which mechanism is better under incomplete information when these two normative standards have equal weights.

How often subjects played the theoretically predicted outcome differed depending on the mechanisms and the informational conditions. In the voluntary contribution mechanism, the ratio of equilibrium strategies that subjects chose under complete information was significantly higher than the ratio chosen under incomplete information, although the ratio under incomplete information increased as periods advanced. Meanwhile, in the pivotal mechanism, the frequency that subjects played dominant strategy equilibria under complete information was not significantly different from the frequency under incomplete information. These observations are consistent with our findings on the comparison of the performances of the two mechanisms: the performance of the voluntary contribution mechanism was higher than the performance of the pivotal mechanism under complete information, but we did not observe the same advantage of the voluntary contribution mechanism over the pivotal mechanism under incomplete information.

The remainder of the paper is organized as follows. Section 2 presents a brief review of the laboratory evidence on the pivotal and voluntary contribution mechanisms. In Section 3, we describe the model of a public project and the definitions of mechanisms. We also propose a new equilibrium concept and provide theoretical prediction results that are useful in examining our experimental observations. Section 4 describes our experimental design. We explain the experimental results in Section 5. Section 6 provides concluding remarks. The appendix contains the proofs of our theoretical results and our experimental instructions.

2 Previous experimental results on the pivotal and voluntary contribution mechanisms

Attiyeh et al. (2000) conducted laboratory experiments on the pivotal mechanism and found that few subjects reported their true values for the public project when only the rule of the pivotal mechanism was explained to groups of 5 or 10 subjects. Less than 10% of the bids were truth-telling in incomplete information settings where each subject knew their own true value but not the others' values in the same group. Kawagoe and Mori (2001) also found that the ratio of the truth-telling bids significantly increased from 16% to 47% when payoff tables were given to subjects in addition to an explanation of the rule of the pivotal mechanism. Moreover, Cason et al. (2006) experimentally compared the Groves mechanism, which is "secure" in the sense that the set of dominant strategy equilibrium outcomes coincides with the set of Nash equilibrium outcomes, with the pivotal mechanism that is not secure. They observed that the ratio of dominant strategies that subjects chose was significantly higher in the secure Groves mechanism than in the non-secure pivotal mechanism.

For the voluntary contribution mechanism with a single unit of public good, laboratory experimental studies found limited support for the theoretical prediction that efficient outcomes are achieved. For example, Marks and Croson (1999) observed that the ratio that decision efficiency was satisfied was 48% under complete information and 54% under incomplete information, and that the ratio of Pareto efficient equilibrium strategies subjects played was 4% under complete information and 6% under incomplete information, even though group contributions converged toward the equilibrium level over time both under complete information and under incomplete information. There are other laboratory experimental studies examining the effects of changing details of the voluntary contribution mechanism such as refund rules, rebate rules, incomplete information on the number of players or the cost of the public project, and simultaneous versus sequential contributions (see Chen (2008)).

Healy (2006) conducted an experimental comparison of five public good mechanisms, the voluntary contribution, proportional taxation, Groves-Ledyard, Walker, and Vickrey-Clarke-Groves (VCG) mechanisms, in the same economic environment with five-person groups under incomplete information. He found that the VCG mechanism was the most efficient among the five mechanisms. In his experimental setting, the level of the public good is chosen from a continuum, and each subject knows their own preference parameters but not the preference parameters of others. On the other hand, the public project choice is binary in our experiment, and we investigate both the incomplete and complete information cases. Moreover, we compare the voluntary contribution mechanism with the pivotal mechanism which is the VCG mechanism with a binary public project from the viewpoints of decision efficiency and individual rationality as well as Pareto efficiency.

3 Preliminaries

3.1 The model

We consider a two-agent economy with a binary public project. Two agents face a decision whether to undertake a public project. Let $y \in \{0,1\}$ be a public project. In our model, $y = 1$ if the public project is undertaken, and $y = 0$ otherwise. The cost of undertaking a public project is $c > 0$. Let $t = (t_1, t_2) \in \mathbb{R}^2$ be a transfer vector, where t_i denotes the payment for agent i. We denote the set of feasible allocations by

$$A = \left\{(y,t) \in \{0,1\} \times \mathbb{R}^2 \colon -(t_1 + t_2) \geq c \cdot y\right\}.$$

For each $i \in \{1,2\}$, let

$$A_i = \{(y,t_i) \in \{0,1\} \times \mathbb{R} \colon -t_i \geq c \cdot y\}$$

be the set of i-feasible allocations.

Each agent $i \in \{1,2\}$ has a quasi-linear preference over $\{0,1\} \times \mathbb{R}$, which is described by the value of the public project $v_i \geq 0$. Denote the set of such values of agent i by V_i. That is, given any $i \in \{1,2\}$ and any $v_i \in V_i$, i's preference is represented by the function $u_i(\,\cdot\,,\,\cdot\,; v_i)\colon \{0,1\} \times \mathbb{R} \to \mathbb{R}$ defined for each $(y, t_i) \in \{0,1\} \times \mathbb{R}$ by

$$u_i(y, t_i; v_i) = v_i \cdot y + \omega_i + t_i,$$

where $\omega_i > 0$ denotes agent i's initial endowment. Let $v = (v_1, v_2) \in V \equiv V_1 \times V_2$ be a value profile.

3.2 Properties of allocations

We introduce some standard properties of allocations. An allocation $(y,t) \in A$ is *Pareto efficient* for $v \in V$ if there exists no allocation $(y', t') \in A$ such that $u_i(y', t'_i; v_i) \geq u_i(y, t_i; v_i)$ for each $i \in \{1,2\}$ with a strict inequality for some $j \in \{1,2\}$. We denote the set of Pareto efficient allocations for v by $P(v)$. The following fact characterizes the set of Pareto efficient allocations.

Fact 1. *Let $v \in V$. An allocation $(y,t) \in A$ is Pareto efficient for v if and only if*

$$u_1(y, t_1; v_1) + u_2(y, t_2; v_2) = \begin{cases} v_1 + v_2 - c + \omega_1 + \omega_2 & \text{if } v_1 + v_2 \geq c \\ \omega_1 + \omega_2 & \text{otherwise.} \end{cases}$$

An allocation $(y,t) \in A$ is *decision efficient* for $v \in V$ if $y \in \arg\max_{y' \in \{0,1\}} (v_1 + v_2) \cdot y' - c \cdot y'$. The following fact states that decision efficiency is weaker than Pareto efficiency.

Fact 2. *Let $v \in V$. If an allocation $(y,t) \in A$ is Pareto efficient for v, then it is decision efficient for v.*

An allocation $(y,t) \in A$ is *individually rational* for $v \in V$ if $u_i(y, t_i; v_i) \geq u_i(y', t'_i; v_i)$ for each $i \in \{1,2\}$ and each $(y', t'_i) \in A_i$. We denote the set of individually rational allocations for v by $I(v)$. The following fact characterizes the set of individually rational allocations.

Fact 3. *Let $v \in V$. An allocation $(y,t) \in A$ is individually rational for v if and only* *if for each* $i \in \{1,2\}$,

$$u_i(y, t_i; v_i) \geq \begin{cases} \omega_i & \text{if } c > v_i \\ v_i - c + \omega_i & \text{otherwise.} \end{cases} \tag{1}$$

The *core* for $v \in V$ is the set of allocations that are Pareto efficient and individually rational for v.[2] We denote the core for v by $C(v)$. That is, $C(v) = P(v) \cap I(v)$ for each $v \in V$.

3.3 Mechanisms

A *mechanism* is a pair $\Gamma = (S_1 \times S_2, g)$, where S_1 and S_2 are message spaces and $g \colon S_1 \times S_2 \to A$ is an *outcome function*. Given $v \in V$, a pair (Γ, v) constitutes a (normal form) game.

We now introduce several equilibrium concepts.

- **Dominant strategy equilibrium.** A strategy profile $(s_1, s_2) \in S_1 \times S_2$ is called a *dominant strategy equilibrium* of the game (Γ, v) if

  (i) for each $s_1' \in S_1$ and each $s_2' \in S_2$, $u_1(g_1(s_1, s_2'); v_1) \geq u_1(g_1(s_1', s_2'); v_1)$; and

  (ii) for each $s_1' \in S_1$ and each $s_2' \in S_2$, $u_2(g_2(s_1', s_2); v_2) \geq u_2(g_2(s_1', s_2'); v_2)$.

- **Nash equilibrium.** A strategy profile $(s_1, s_2) \in S_1 \times S_2$ is called a *Nash equilibrium* of the game (Γ, v) if

  (i) for each $s_1' \in S_1$, $u_1(g_1(s_1, s_2); v_1) \geq u_1(g_1(s_1', s_2); v_1)$; and

  (ii) for each $s_2' \in S_2$, $u_2(g_2(s_1, s_2); v_2) \geq u_2(g_2(s_1, s_2'); v_2)$.

- **Undominated Nash equilibrium.** Given any $i \in \{1,2\}$, we say that an agent i's strategy $s_i \in S_i$ is *weakly dominated* in the game (Γ, v) if there exists $s_i' \in S_i$ such that for each $s_j \in S_j$, $u_i(g_i(s_i', s_j); v_i) \geq u_i(g_i(s); v_i)$ with a strict inequality for some $s_j' \in S_j$. Denote the set of i's strategies that are not weakly dominated in the game (Γ, v) by $U(S_i; (\Gamma, v))$. A strategy profile $(s_1, s_2) \in S_1 \times S_2$ is called an *undominated Nash equilibrium* of the game

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[2] This definition depends on the number of agents. When there are three or more agents, the core is generally smaller than the set of allocations satisfying Pareto efficiency and individual rationality.

(Γ, v) if (i) it is a Nash equilibrium of (Γ, v) and (ii) $s_i \in U(S_i; (\Gamma, v))$ for each $i \in \{1, 2\}$.

- **Twice iteratively undominated Nash equilibrium.** Let $U(S; (\Gamma, v)) \equiv U(S_1; (\Gamma, v)) \times U(S_2; (\Gamma, v))$ and $\Gamma(U) \equiv (U(S; (\Gamma, v)), g)$. A strategy profile $(s_1, s_2) \in S_1 \times S_2$ is called a *twice iteratively undominated Nash equilibrium* of the game (Γ, v) if it is an undominated Nash equilibrium of the game $(\Gamma(U), v)$. Let $\mathcal{E}(\Gamma, v)$ and $\mathcal{A}(\Gamma, v)$ be the set of twice iteratively undominated Nash equilibria for (Γ, v) and the set of twice iteratively undominated Nash equilibrium allocations for (Γ, v), respectively.

3.3.1 The voluntary contribution mechanism

We consider two mechanisms that play central roles in the literature. Our first mechanism is the following simple mechanism that is widely used in practice: each agent offers a voluntary contribution. If the sum of the contributions covers the cost of the public project, it is undertaken; otherwise, the contributions are returned. Formally:

The voluntary contribution mechanism, $\boldsymbol{\Gamma^{\mathrm{VC}} = (S_1^{\mathrm{VC}} \times S_2^{\mathrm{VC}}, g^{\mathrm{VC}})}$. For each $i \in \{1, 2\}$, $S_i^{\mathrm{VC}} = [0, \omega_i]$; and for each $s \in S_1^{\mathrm{VC}} \times S_2^{\mathrm{VC}}$,

$$g^{\mathrm{VC}}(s) = (y^{\mathrm{VC}}(s), (t_1^{\mathrm{VC}}(s), t_2^{\mathrm{VC}}(s))) = \begin{cases} (0, (0, 0)) & \text{if } s_1 + s_2 < c \\ (1, (-s_1, -s_2)) & \text{otherwise.} \end{cases}$$

This subsection now makes the following assumption:

A1. For each $v \in V$ and each $i \in \{1, 2\}$, $\omega_i \geq c > v_i$.

A1 says that each agent's initial endowment exceeds the cost of undertaking a public project, and each agent's value is less than the cost of undertaking a public project. A1 also says that no agent would like to contribute more than their initial endowment.[3] We conduct laboratory experiments in the environments that satisfy A1.

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[3] Instead of A1, Bagnoli and Lipman (1989) used the following weaker assumption: $\omega_1 + \omega_2 > c$ and for each $v \in V$ and each $i \in \{1, 2\}$, $\omega_i > v_i$.

Under A1, Facts 1 and 3 together imply that for each $v \in V$,

$$C(v) = \begin{cases} \left\{ (1,t) \in A \colon \begin{array}{c} 0 \leq -t_1 \leq v_1,\ 0 \leq -t_2 \leq v_2, \text{ and} \\ -(t_1 + t_2) = c \end{array} \right\} & \text{if } v_1 + v_2 > c \\ \{(0,(0,0)), (1,(-v_1,-v_2))\} & \text{if } v_1 + v_2 = c \\ \{(0,(0,0))\} & \text{if } v_1 + v_2 < c. \end{cases}$$

The next proposition states that the twice iteratively undominated Nash equilibrium allocations of the voluntary contribution mechanism are in the core for true preferences.[4]

Proposition 1. *Assume A1. Let $v \in V$. Then,*

$$\mathcal{A}(\Gamma^{\text{VC}}, v) \subseteq C(v).$$

The voluntary contribution mechanism is simple and, thus, it can be easily implemented in practice. However, to obtain allocations that are in the core via the voluntary contribution mechanism, agents are required to have a higher degree of rationality, and the mechanism designer must understand that a value profile is common knowledge between agents.

Remark 1. Our notion of the twice iteratively undominated Nash equilibrium is different from the notion of iteratively undominated strategies. The reason we do not employ the notion of iteratively undominated strategies is that the voluntary contribution mechanism cannot implement the core (and any sub-correspondence of the core) in iteratively undominated strategies. See Appendix B for a more detailed discussion. ♢

Remark 2. To show that the voluntary contribution mechanism implements the core, Bagnoli and Lipman (1989) used another refinement of Nash equilibrium called "undominated perfect equilibrium" that applies the notion of (trembling-hand) perfection to the game after eliminating dominated strategies of the original game. However, they did not directly apply the notion of perfection to the infinite game

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[4] This is equivalent to stating that the voluntary contribution mechanism implements a sub-correspondence of the core in twice iteratively undominated Nash equilibria. Moreover, we could argue that the voluntary contribution mechanism "almost" fully implements the core in twice iteratively undominated Nash equilibria because the following facts hold: for each $v \in V$, (i) if $v_1 + v_2 > c$, then $C(v) \setminus \mathcal{A}(\Gamma^{\text{VC}}, v) = \{(1,(-v_1, v_1 - c)), (1,(v_2 - c, -v_2))\}$; (ii) if $v_1 + v_2 = c$, then $C(v) \setminus \mathcal{A}(\Gamma^{\text{VC}}, v) = \{(1,(-v_1,-v_2))\}$; and (iii) if $v_1 + v_2 < c$, then $C(v) = \mathcal{A}(\Gamma^{\text{VC}}, v)$.

associated with the voluntary contribution mechanism.[5] Instead, they defined an undominated perfect equilibrium in the infinite game associated with the voluntary contribution mechanism as the limit of some sequence of undominated perfect equilibria of approximating finite versions of the voluntary contribution mechanism. Thus, it is an interesting open question whether a similar implementation result can be obtained when perfection is directly defined on the infinite game associated with the voluntary contribution mechanism. ♢

3.3.2 The pivotal mechanism

Our second mechanism determines the allocation as follows: each agent $i \in \{1, 2\}$ reports their value v_i of the public project simultaneously. If $v_1 + v_2 \geq c$, then the public project is undertaken, and it is not undertaken otherwise. If the public project is undertaken, then both agents pay for an equal share of the project's cost. Moreover, whether the public project is undertaken or not, each agent $i \in \{1, 2\}$ must pay the *pivotal tax* p_i defined as follows:

$$p_i = \begin{cases} -\left(v_j - \dfrac{c}{2}\right) & \text{if } v_1 + v_2 \geq c \text{ and } v_j - \dfrac{c}{2} < 0 \\ v_j - \dfrac{c}{2} & \text{if } v_1 + v_2 < c \text{ and } v_j - \dfrac{c}{2} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The pivotal tax is the actual net benefit the other agent enjoys minus the maximal net benefit the agent can enjoy under the equal share cost. Thus, the pivotal tax is the social cost of considering agent i's preference. Formally:

The pivotal mechanism, $\boldsymbol{\Gamma^{\mathrm{P}} = (S_1^{\mathrm{P}} \times S_2^{\mathrm{P}}, g^{\mathrm{P}})}$. For each $i \in \{1, 2\}$, $S_i^{\mathrm{P}} = V_i$; and for each $v \in V = S_1^{\mathrm{P}} \times S_2^{\mathrm{P}}$, we denote $g^{\mathrm{P}}(v) = (y^{\mathrm{P}}(v), (t_1^{\mathrm{P}}(v), t_2^{\mathrm{P}}(v)))$, where

$$\begin{aligned} y^{\mathrm{P}}(v) &\in \arg \max_{y \in \{0,1\}} (v_1 + v_2) \cdot y + c \cdot y, \\ t_1^{\mathrm{P}}(v) &= -\frac{c \cdot y^{\mathrm{P}}(v)}{2} + \left(v_2 \cdot y^{\mathrm{P}}(v) - \frac{c \cdot y^{\mathrm{P}}(v)}{2}\right) - \max_{y \in \{0,1\}} \left(\frac{c \cdot y}{2} + v_2 \cdot y\right), \\ t_2^{\mathrm{P}}(v) &= -\frac{c \cdot y^{\mathrm{P}}(v)}{2} + \left(v_1 \cdot y^{\mathrm{P}}(v) - \frac{c \cdot y^{\mathrm{P}}(v)}{2}\right) - \max_{y \in \{0,1\}} \left(\frac{c \cdot y}{2} + v_1 \cdot y\right). \end{aligned}$$

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[5] Recently, Carbonell-Nicolau (2011) provided a notion of perfection in infinite games. By invoking his notion of perfection, he examined the existence of undominated perfect equilibrium in the infinite game associated with the voluntary contribution mechanism. However, it is an open question whether a similar implementation result holds for his notion of perfection.

The pivotal mechanism might appear less simple than the voluntary contribution mechanism. However, it is well known that under the pivotal mechanism, truth-telling is a dominant strategy for everyone. Therefore, in contrast to the voluntary contribution mechanism, in the pivotal mechanism, agents need not care about the exact value of the other agent when considering their own action. Moreover, although the dominant strategy equilibrium allocations of this mechanism may not be Pareto efficient or individually rational, they are always decision efficient.

4 Experimental design

4.1 Design

Our experiment studies the voluntary contribution mechanism and the pivotal mechanism both under complete information and under incomplete information. It consists of four treatments:

(i) **Treatment VC:** the voluntary contribution mechanism under complete information

(ii) **Treatment VI:** the voluntary contribution mechanism under incomplete information

(iii) **Treatment PC:** the pivotal mechanism under complete information

(iv) **Treatment PI:** the pivotal mechanism under incomplete information

The public project problem to be solved is the same for all four treatments. There are two types of agents, 1 and 2. The initial endowment vector of the private good is given by $(\omega_1, \omega_2) = (20, 20)$. The true value profile (v_1, v_2) is equal to $(9, 13)$. The cost of undertaking the public project is $c = 20$. By decision efficiency introduced in Section 3, the public project should be undertaken since $v_1 + v_2 - c = 2 > 0$. Note that this environment satisfies A1 introduced in Section 3.

4.1.1 Treatment VC

Treatment VC implements the voluntary contribution mechanism for a two-agent group under complete information. Let the strategy space of each type be the set of integers from 0 to 20. According to the rules of the voluntary contribution mechanism explained in Section 3, we can construct the payoff matrices of Types

1 and 2. The payoff tables that we employ in Treatment VC are Tables 1 and 2 whose structures are the same as the original payoff tables with the exception that a linear transformation of the valuation functions is employed: $10u_i - 40$ for each type $i \in \{1, 2\}$.

Table 1 (resp. Table 2) is a payoff matrix of Type 1 (resp. Type 2) with both types' payoffs displayed: the lower left-hand number is Type 1's (resp. Type 2's) payoff, and the upper right-hand number is Type 2's (resp. Type 1's) payoff in each cell. Table 1 also specifies the Nash equilibria and the unique twice iteratively undominated Nash equilibrium.[6] In Table 1, there are three "good" Nash equilibria, $(s_1, s_2) \in \{(7, 13), (8, 12), (9, 11)\}$, in the sense that the public project is undertaken and, hence, decision efficiency is met, whereas there are 96 "bad" Nash equilibria, $(s_1, s_2) \in \{(0, 0), \ldots, (0, 11), \ldots, (7, 0), \ldots, (7, 11)\}$, in the sense that the public project is not undertaken and, thus, decision efficiency is not satisfied. Among these Nash equilibria, only the pair $(s_1, s_2) = (8, 12)$ is a twice iteratively undominated Nash equilibrium. Note that Type 1's strategies $s_1 \in \{9, 10, \ldots, 20\}$ are weakly dominated by $s_1 = 8$, whereas $s_1 \in \{0, 1, \ldots, 7\}$ are undominated strategies that are not weakly dominated by any strategy. Similarly, Type 2's strategies $s_2 \in \{13, \ldots, 20\}$ are weakly dominated by $s_2 = 12$, whereas $s_2 \in \{0, 1, \ldots, 11\}$ are undominated strategies. Eliminating these weakly dominated strategies of two players from Table 1 of the original 21×21 payoff table, we have Table 3 that is the 8×12 payoff matrix consisting of eight strategies of Type 1, $s_1 \in \{0, 1, \ldots, 7\}$ and 12 strategies of Type 2, $s_2 \in \{0, 1, \ldots, 11\}$. In Table 3, there is one "good" Nash equilibria, $(s_1, s_2) = (8, 12)$, whereas there are 96 "bad" Nash equilibria, $(s_1, s_2) \in \{(0, 0), \ldots, (0, 11), \ldots, (7, 0), \ldots, (7, 11)\}$. Notice that Type 1's strategies $s_1 \in \{0, 1, \ldots, 7\}$ are weakly dominated by $s_1 = 8$, and Type 2's strategies $s_2 \in \{0, 1, \ldots, 11\}$ are weakly dominated by $s_2 = 12$ in Table 3. Thus, $(8, 12)$ is a unique undominated Nash equilibrium in Table 3 and a unique twice iteratively undominated Nash equilibrium in Table 1.[7]

Moreover, this equilibrium payoff vector $(170, 170)$ at $(8, 12)$ is Pareto efficient

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[6] In the payoff table that we distributed to subjects, there was no tag and highlighting indicating the equilibria as in Table 1.

[7] Additionally, $(8, 12)$ is a unique pair of strategies surviving after a twice iterated elimination of weakly dominated strategies in Table 1. In other words, the unique undominated Nash equilibrium coincides with the unique pair of strategies surviving after twice iterated elimination of weakly dominated strategies under our experimental setting. In general, however, the set of twice iteratively undominated Nash equilibria may differ from the set of pairs of strategies surviving after twice iterated elimination of weakly dominated strategies. We need the concept of twice iteratively undominated Nash equilibria to obtain the core implementation result in Proposition 1. See Appendix B for more details.

The number which the other person chooses (Type 2)

V1

The number which you choose (Type 1)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	90 250
1	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	100 240	90 240
2	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	110 230	100 230	90 230
3	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	120 220	110 220	100 220	90 220
4	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	130 210	120 210	110 210	100 210	90 210
5	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	140 200	130 200	120 200	110 200	100 200	90 200
6	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	150 190	140 190	130 190	120 190	110 190	100 190	90 190
7	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 180	150 180	140 180	130 180	120 180	110 180	100 180	90 180
8	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	170 170	160 170	150 170	140 170	130 170	120 170	110 170	100 170	90 170
9	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	180 160	170 160	160 160	150 160	140 160	130 160	120 160	110 160	100 160	90 160
10	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	190 150	180 150	170 150	160 150	150 150	140 150	130 150	120 150	110 150	100 150	90 150
11	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	200 140	190 140	180 140	170 140	160 140	150 140	140 140	130 140	120 140	110 140	100 140	90 140
12	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	210 130	200 130	190 130	180 130	170 130	160 130	150 130	140 130	130 130	120 130	110 130	100 130	90 130
13	160 160	160 160	160 160	160 160	160 160	160 160	160 160	220 120	210 120	200 120	190 120	180 120	170 120	160 120	150 120	140 120	130 120	120 120	110 120	100 120	90 120
14	160 160	160 160	160 160	160 160	160 160	160 160	230 110	220 110	210 110	200 110	190 110	180 110	170 110	160 110	150 110	140 110	130 110	120 110	110 110	100 110	90 110
15	160 160	160 160	160 160	160 160	160 160	240 100	230 100	220 100	210 100	200 100	190 100	180 100	170 100	160 100	150 100	140 100	130 100	120 100	110 100	100 100	90 100
16	160 160	160 160	160 160	160 160	250 90	240 90	230 90	220 90	210 90	200 90	190 90	180 90	170 90	160 90	150 90	140 90	130 90	120 90	110 90	100 90	90 90
17	160 160	160 160	160 160	260 80	250 80	240 80	230 80	220 80	210 80	200 80	190 80	180 80	170 80	160 80	150 80	140 80	130 80	120 80	110 80	100 80	90 80
18	160 160	160 160	270 70	260 70	250 70	240 70	230 70	220 70	210 70	200 70	190 70	180 70	170 70	160 70	150 70	140 70	130 70	120 70	110 70	100 70	90 70
19	160 160	280 60	270 60	260 60	250 60	240 60	230 60	220 60	210 60	200 60	190 60	180 60	170 60	160 60	150 60	140 60	130 60	120 60	110 60	100 60	90 60
20	290 50	280 50	270 50	260 50	250 50	240 50	230 50	220 50	210 50	200 50	190 50	180 50	170 50	160 50	150 50	140 50	130 50	120 50	110 50	100 50	90 50

The other's payoff

Your Payoff

▨ Bad Nash Equilbria

▨ Good Nash Equilbria

▨ Twice Iteratively Undominated Nash Equilibrium = Undominated Nash Equilibrium in the 8x12 Payoff Table Obtained by Eliminating Weakly Dominated Strategies (Table 3).

Table 1: Payoff table of Type 1 in Treatment VC.

The number which the other person chooses (Type 1)

V2

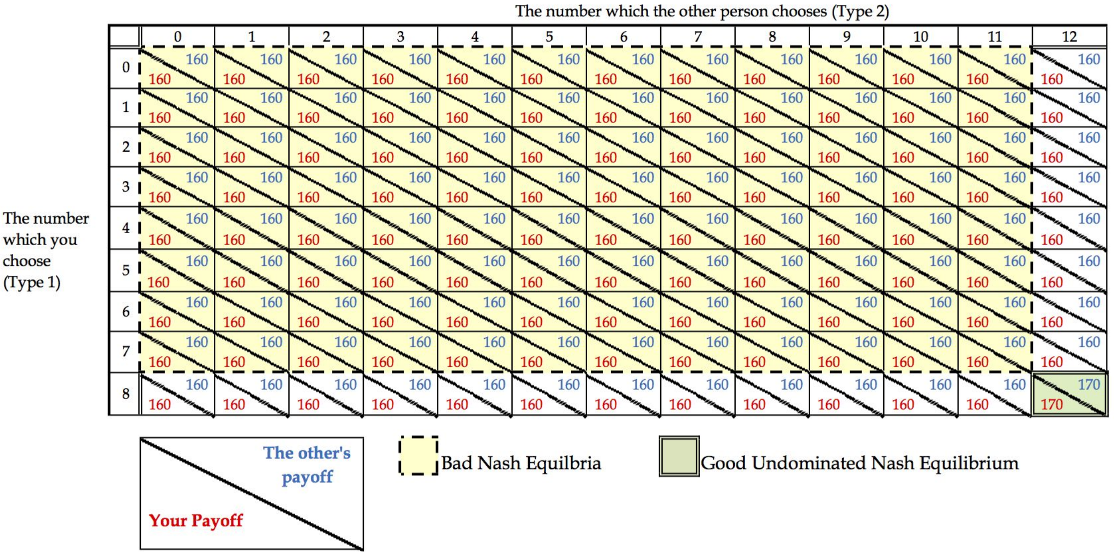
The number which you choose (Type 2)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	50 290
1	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	60 280	50 280
2	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	70 270	60 270	50 270
3	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	80 260	70 260	60 260	50 260
4	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	90 250	80 250	70 250	60 250	50 250
5	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	100 240	90 240	80 240	70 240	60 240	50 240
6	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	110 230	100 230	90 230	80 230	70 230	60 230	50 230
7	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	120 220	110 220	100 220	90 220	80 220	70 220	60 220	50 220
8	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	130 210	120 210	110 210	100 210	90 210	80 210	70 210	60 210	50 210
9	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	140 200	130 200	120 200	110 200	100 200	90 200	80 200	70 200	60 200	50 200
10	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	150 190	140 190	130 190	120 190	110 190	100 190	90 190	80 190	70 190	60 190	50 190
11	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 180	150 180	140 180	130 180	120 180	110 180	100 180	90 180	80 180	70 180	60 180	50 180
12	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	170 170	160 170	150 170	140 170	130 170	120 170	110 170	100 170	90 170	80 170	70 170	60 170	50 170
13	160 160	160 160	160 160	160 160	160 160	160 160	160 160	180 160	170 160	160 160	150 160	140 160	130 160	120 160	110 160	100 160	90 160	80 160	70 160	60 160	50 160
14	160 160	160 160	160 160	160 160	160 160	160 160	190 150	180 150	170 150	160 150	150 150	140 150	130 150	120 150	110 150	100 150	90 150	80 150	70 150	60 150	50 150
15	160 160	160 160	160 160	160 160	160 160	200 140	190 140	180 140	170 140	160 140	150 140	140 140	130 140	120 140	110 140	100 140	90 140	80 140	70 140	60 140	50 140
16	160 160	160 160	160 160	160 160	210 130	200 130	190 130	180 130	170 130	160 130	150 130	140 130	130 130	120 130	110 130	100 130	90 130	80 130	70 130	60 130	50 130
17	160 160	160 160	160 160	220 120	210 120	200 120	190 120	180 120	170 120	160 120	150 120	140 120	130 120	120 120	110 120	100 120	90 120	80 120	70 120	60 120	50 120
18	160 160	160 160	230 110	220 110	210 110	200 110	190 110	180 110	170 110	160 110	150 110	140 110	130 110	120 110	110 110	100 110	90 110	80 110	70 110	60 110	50 110
19	160 160	240 100	230 100	220 100	210 100	200 100	190 100	180 100	170 100	160 100	150 100	140 100	130 100	120 100	110 100	100 100	90 100	80 100	70 100	60 100	50 100
20	250 90	240 90	230 90	220 90	210 90	200 90	190 90	180 90	170 90	160 90	150 90	140 90	130 90	120 90	110 90	100 90	90 90	80 90	70 90	60 90	50 90

The other's payoff

Your Payoff

Table 2: Payoff table of Type 2 in Treatment VC.



		The number which the other person chooses (Type 2)												
		0	1	2	3	4	5	6	7	8	9	10	11	12
The number which you choose (Type 1)	0	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	1	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	2	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	3	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	4	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	5	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	6	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	7	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160
	8	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	170 170

Table 3: Payoff table obtained by eliminating the weakly dominated strategies of two players from Table 1.

and individually rational and, hence, it is a core payoff vector (Proposition 1). The other good Nash equilibrium payoff vectors in Table 1, $(180, 160)$ and $(160, 180)$, are also Pareto efficient and individually rational, and they are core payoff vectors. All bad Nash equilibrium payoff vectors in Table 1 are the same, $(160, 160)$, and they are individually rational but not Pareto efficient.

4.1.2 Treatment VI

Treatment VI implementing the voluntary contribution mechanism under incomplete information is the same as Treatment VC except for the payoff tables. In Treatment VI, we used the payoff tables deleting the payoffs of others from Tables 1 and 2 so that the payoff table of each type indicates their own payoffs only.

4.1.3 Treatment PC

Treatment PC implements the pivotal mechanism for a two-agent group under complete information. Let the strategy space of each type be the set of integers from 0 to 20. According to the rules of the pivotal mechanism described in Section 3, we can construct the payoff tables of Types 1 and 2. The payoff tables that we use in Treatment PC are Tables 4 and 5 whose structures are the same as the original payoff tables except that a linear transformation of the utility functions is employed: $10u_i - 40$ for each type $i \in \{1, 2\}$.

Table 4 (resp. Table 5) is a payoff table of Type 1 (resp. Type 2) with both types' payoffs displayed. Table 4 also specifies the dominant strategy equilibria and the other Nash equilibria.[8] Type 1's dominant strategies are 8 and 9, and Type 2's dominant strategies are 12 and 13. The two dominant strategies are equivalent for each type in that their own payoffs are identical for every possible strategy played by the other type. In this sense, there is an essentially unique dominant strategy for each type. However, the payoffs of Type 2 are different depending on Type 1's choices: Type 2's payoff is 170 when Type 1 selects 8 and Type 2 chooses 12 or 13, whereas Type 2's payoff is 180 when Type 1 selects 9 and Type 2 chooses 12 or 13.

There is a huge set of Nash equilibria in Table 4.[9] The lower-right region of Nash equilibria is "good" in the sense that the public project is undertaken and, hence, decision efficiency is satisfied. The upper-left region of Nash equilibria is "bad" in the sense that the public project is not undertaken and, thus, decision efficiency is not satisfied. The number of good Nash equilibria is 137, whereas the number of bad Nash equilibria is 96. Implementation is clearly not secure in the sense of Cason et al. (2006).

Notice that the payoff vector (u_1, u_2) is Pareto efficient if $u_1 + u_2 = 340$, and it is individually rational if $u_1 \geq 160$ and $u_2 \geq 160$ in the current environment. The dominant strategy equilibrium payoffs, $(150, 170)$ and $(150, 180)$, are neither Pareto efficient nor individually rational. This is because the pivotal mechanism generates a wasteful budget surplus, that is, the tax revenue exceeds the cost of undertaking the public project. It is easy to check that the ratio of Pareto efficient payoffs among the good Nash equilibrium payoffs is 80.3% (= 110/137), but no good Nash equilibrium payoff is individually rational. The ratio of individually rational payoffs among the bad Nash equilibrium payoffs is 91.7% (= 88/96), but no bad Nash equilibrium payoff is Pareto efficient.

Although no Nash equilibrium payoff vector belongs to the core, the pivotal

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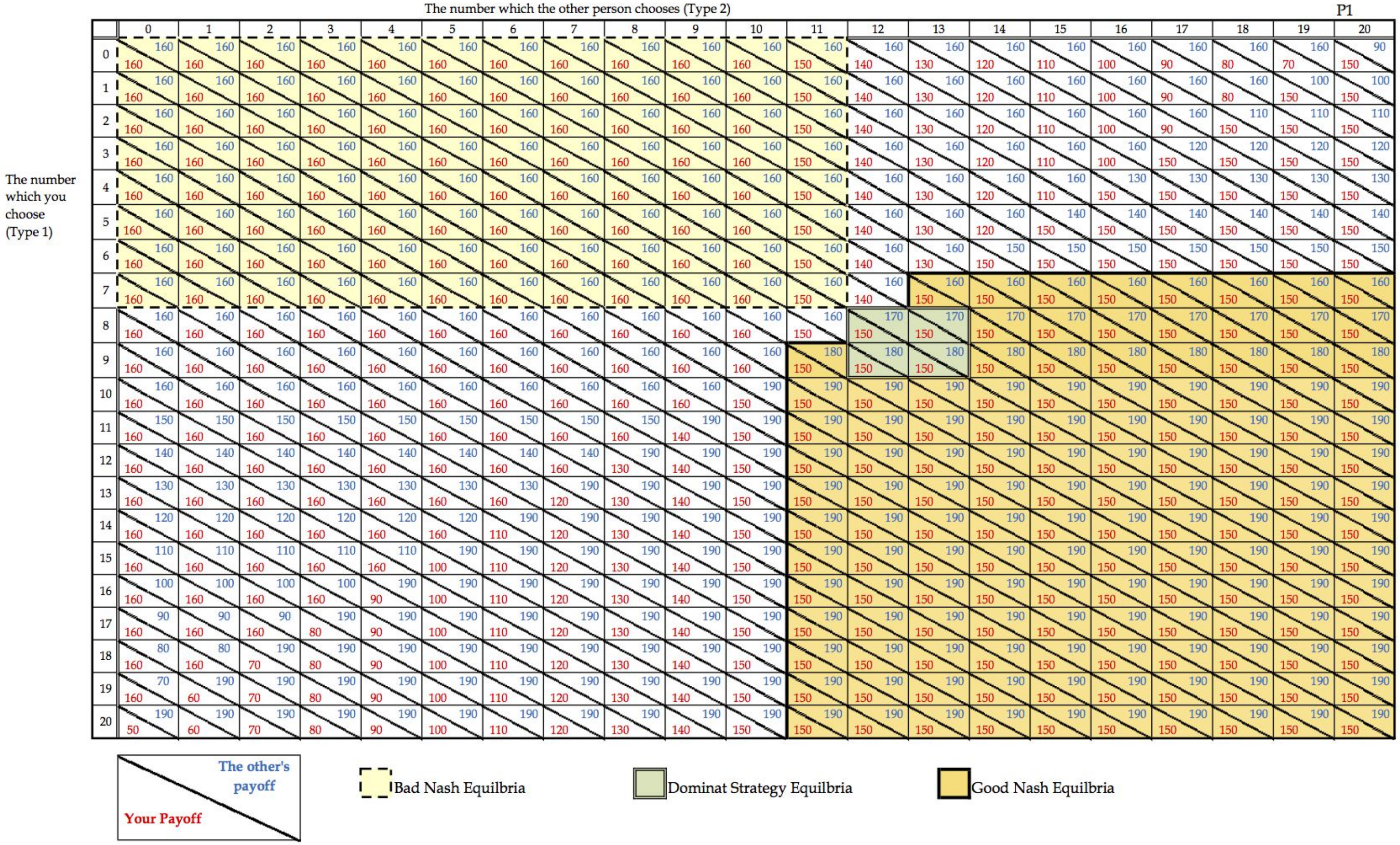
[8] Again, in the payoff table that we actually distributed to subjects, there was no tag and highlighting indicating the equilibria, as in Table 4 .

[9] This is because the best response function of each type has a "flat" structure. For instance, (i) when Type 2 selects 7, the payoffs of Type 1 are "high" (160) if Type 1 chooses less than or equal to 12, and those are "low" (120) otherwise; (ii) when Type 2 chooses 11, the payoffs of Type 1 are the same (150) for all Type 1's strategies; and (iii) when Type 2 selects 16, the payoffs of Type 1 are "low" (100) if Type 1 chooses less than or equal to 3, and Type 1's payoffs are "high" (150) otherwise. In other words, given each strategy of the other type, either a) the payoffs of each type are divided into just two "tiers": a "high" payoff obtained by choosing "good" strategies and a "low" payoff by "bad" strategies, or b) the payoffs are the same for all strategies.



The number which the other person chooses (Type 2)

P1

The number which you choose (Type 1)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	160 80	160 70	90 150
1	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	160 80	100 150	100 150
2	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	110 150	110 150	110 150
3	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	120 150	120 150	120 150	120 150
4	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	130 150	130 150	130 150	130 150	130 150
5	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	140 150	140 150	140 150	140 150	140 150	140 150
6	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	150 150	150 150	150 150	150 150	150 150	150 150	150 150
7	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 150	160 150	160 150	160 150	160 150	160 150	160 150	160 150
8	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	170 150	170 150	170 150	170 150	170 150	170 150	170 150	170 150	170 150
9	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	180 150	180 150	180 150	180 150	180 150	180 150	180 150	180 150	180 150	180 150
10	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
11	150 160	150 160	150 160	150 160	150 160	150 160	150 160	150 160	150 160	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
12	140 160	140 160	140 160	140 160	140 160	140 160	140 160	140 160	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
13	130 160	130 160	130 160	130 160	130 160	130 160	130 160	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
14	120 160	120 160	120 160	120 160	120 160	120 160	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
15	110 160	110 160	110 160	110 160	110 160	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
16	100 160	100 160	100 160	100 160	190 90	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
17	90 160	90 160	90 160	190 80	190 90	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
18	80 160	80 160	190 70	190 80	190 90	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
19	70 160	190 60	190 70	190 80	190 90	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150
20	190 50	190 60	190 70	190 80	190 90	190 100	190 110	190 120	190 130	190 140	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150	190 150

The other's payoff

Your Payoff

Bad Nash Equilbria

Dominat Strategy Equilbria

Good Nash Equilbria

Table 4: Payoff table of Type 1 in Treatment PC.

The number which the other person chooses (Type 1)

P2

The number which you choose (Type 2)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	160 80	160 70	50 190
1	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	160 80	60 190	60 190
2	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	160 90	70 190	70 190	70 190
3	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	160 100	80 190	80 190	80 190	80 190
4	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	160 110	90 190	90 190	90 190	90 190	90 190
5	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	160 120	100 190	100 190	100 190	100 190	100 190	100 190
6	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	160 130	110 190	110 190	110 190	110 190	110 190	110 190	110 190
7	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	160 140	120 190	120 190	120 190	120 190	120 190	120 190	120 190	120 190
8	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 150	130 190	130 190	130 190	130 190	130 190	130 190	130 190	130 190	130 190
9	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	140 190	140 190	140 190	140 190	140 190	140 190	140 190	140 190	140 190	140 190
10	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	160 160	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
11	150 160	150 160	150 160	150 160	150 160	150 160	150 160	150 160	150 160	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
12	140 160	140 160	140 160	140 160	140 160	140 160	140 160	140 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
13	130 160	130 160	130 160	130 160	130 160	130 160	130 160	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
14	120 160	120 160	120 160	120 160	120 160	120 160	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
15	110 160	110 160	110 160	110 160	110 160	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
16	100 160	100 160	100 160	100 160	150 130	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
17	90 160	90 160	90 160	150 120	150 130	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
18	80 160	80 160	150 110	150 120	150 130	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
19	70 160	150 100	150 110	150 120	150 130	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190
20	150 90	150 100	150 110	150 120	150 130	150 140	150 150	150 160	150 170	150 180	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190	150 190

The other's payoff

Your Payoff

Table 5: Payoff table of Type 2 in Treatment PC.

mechanism is designed specifically to achieve decision efficiency rather than Pareto efficiency or individual rationality. Since decision efficiency has played a central role in the literature, we check whether the outcome is consistent with decision efficiency in our experiment.

4.1.4 Treatment PI

Treatment PI implementing the pivotal mechanism under incomplete information, is the same as Treatment PC except for the payoff tables. In Treatment PI, we use the payoff tables deleting the payoffs of others from Tables 4 and 5 so that the payoff table of each type indicates their own payoffs only.

4.2 Procedures

We conducted two sessions in each of the four treatments at Tokyo Institute of Technology during January and June of 2013 and July of 2014. Twenty subjects participated in each session (160 separate subjects in total). We recruited the student subjects by campus-wide advertisement. These students were told that there would be an opportunity to earn money in a research experiment. None of them had prior experience in a public project experiment. No subject attended more than one session. Each session took approximately two hours to complete. The mean payoff per subject was \$32.57 (\$1 = 100 yen) in Treatment PC, \$32.54 in Treatment PI, \$33.56 in Treatment VC, and \$32.31 in Treatment VI.

In each session, 20 subjects were seated at computer stations that were separated with visual partitions in the Experimental Economics Laboratory at Tokyo Institute of Technology. We made 10 pairs out of 20 subjects and conducted 20 periods. In every period, each of Type 1 subjects was paired with one of Type 2 subjects. The pairings were anonymous and were determined in advance by experimenters to pair the same two subjects just two times. Each subject was informed that the person the subject was paired with was randomly chosen by the experimenters and that the person the subject was paired with changed over the 20 periods. Each subject could not know which person the subject was paired with at each period.

Each subject received written instructions, a record sheet, and a payoff table. Each subject chose an integer number between 0 and 20 by looking at their own payoff table only. No subject knew the payoff table of the other type in Treatments PI and VI under incomplete information. In contrast, in each subject's payoff table, both their own payoffs and the payoffs of the other type were shown in Treatments

PC and VC under complete information.

After deciding which number to choose, each subject typed in that number into the computer. We used the z-Tree program (Urs Fischbacher, 2007). After the calculation of payoffs, the following information was displayed on each subject's computer screen:

- **Treatments PC and VC (complete information):** the subject's chosen number, the other's chosen number, the subject's own payoff, and the other's payoff.
- **Treatments PI and VI (incomplete information):** the subject's chosen number, the other's chosen number, and the subject's own payoff (but not the other's payoff).

Each subject was asked to fill out these values as well as the reasons why they had chosen a particular number. These steps were repeated for 20 periods. No information or decisions regarding the other pairs were shown on the computer screen. No communication among the subjects was allowed, and we declared that the experiment would be stopped if we observed any communication among the subjects. This did not happen.

Before the real 20 periods, the subjects had an opportunity to practice in two periods using a payoff table that differed from the table employed in the actual experiment. In these non-monetary periods, the numbers to be chosen were decided in advance by the experimenters. We allowed the subjects 10 minutes to examine the payoff table before the real periods started.

5 Results

Since each period had 20 pairs of players and each session had 20 periods, there were 400 pairs of data. We denote each pair by (s_1, s_2), where s_i is a strategy chosen by a subject of Type $i \in \{1, 2\}$.

5.1 Complete information

5.1.1 Treatment VC

Figure 1 shows the frequency of distribution of all data in Treatment VC. The maximum frequency pair was the unique twice iteratively undominated Nash equilibrium

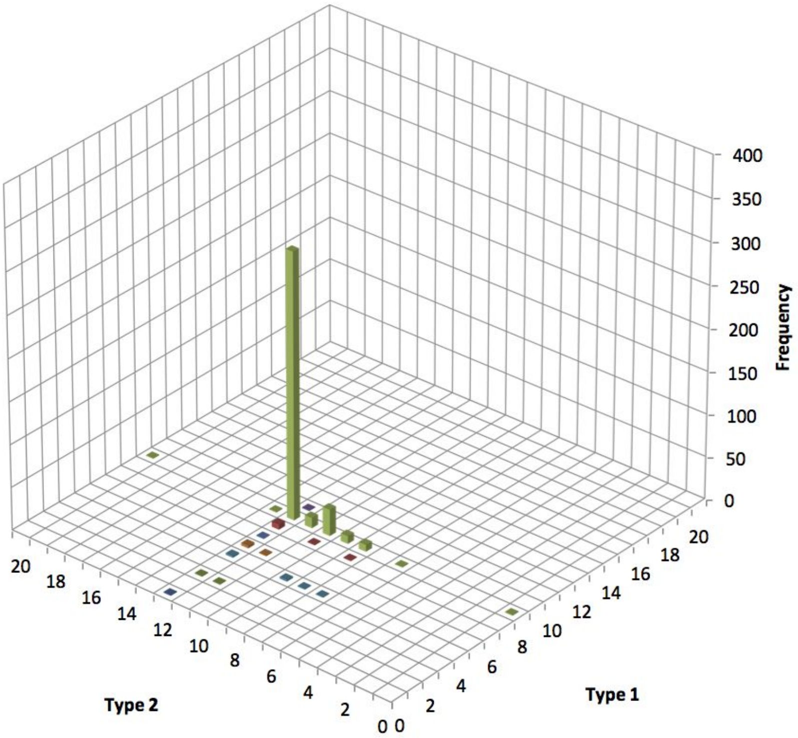


Figure 1: All pairs choices in Treatment VC.

(8, 12) with 313 pairs out of 400, the second frequency pair was (8, 10) with 31 pairs, the third frequency pair was (8, 11) with 11 pairs, and the fourth frequency pair was (7, 12) with 6 pairs. The total frequency of Nash equilibria other than (8, 12) was 8. All of them were bad Nash equilibria that failed to achieve decision efficiency. The total frequency of strategy profiles that achieved decision efficiency was 316.

The maximum frequency strategy chosen by Type 1 subjects was $s_1 = 8$ with 377 choices out of 400, the second maximum frequency strategy was $s_1 = 7$ with 8 choices, and the third maximum frequency strategy was $s_1 = 4$ with 6 choices. The maximum frequency strategy chosen by Type 2 subjects was $s_2 = 12$ with 328 choices out of 400, the second was $s_2 = 10$ with 32 choices, and the third was $s_1 = 11$ with 14 choices.

Figure 2 displays the rates that Type 1 and Type 2 subjects chose $s_1 = 8$ and $s_2 = 12$, respectively. After period 3, the ratios of Type 1 subjects who chose $s_1 = 8$ were at least 90%. The ratios of Type 2 subjects who chose $s_2 = 12$ were at least 80% after period 4. Figure 2 also demonstrates the rates that pairs of subjects played the unique twice iteratively undominated Nash equilibrium (8, 12) for all periods. These equilibrium rates were at least 70% after period 4. By conducting the Wilcoxon signed rank tests period by period, we found that Type 1 subjects' median choice was not significantly different from 8 in 18 out of 20 periods (periods 3–20 at the 5% significance level) and that Type 2 subjects' median choice was not significantly different from 12 in 18 periods (periods 2 and 4–20 at the 5% significance level).

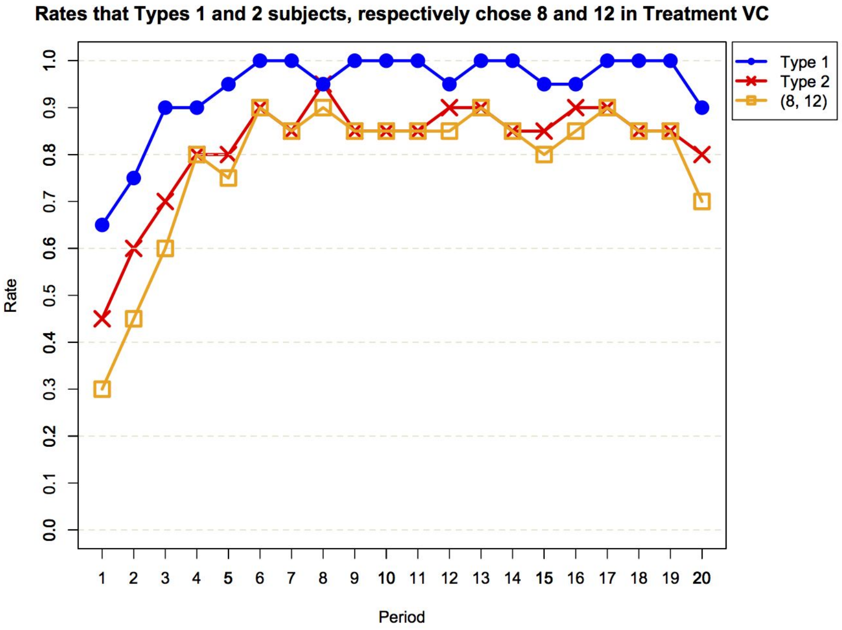


Figure 2: Rates that Types 1 and 2 subjects, respectively, chose $s_1 = 8$ and $s_2 = 12$ in Treatment VC.

These results are summarized by the following observations.

Observation 1.

(i) *The frequency of the unique twice iteratively undominated Nash equilibrium was 78% (= 313/400) across all periods in Treatment VC.*

(ii) *The frequency that Type 1 subjects chose* $s_1 = 8$ *was 94%(= 377/400) across all periods in Treatment VC.*

(iii) *The frequency that Type 2 subjects chose* $s_2 = 12$ *was 82%(= 328/400) across all periods in Treatment VC.*

5.1.2 Treatment PC

Figure 3 shows the frequency of distribution of all data in Treatment PC. The maximum frequency pair was $(8, 13)$ with 95 pairs out of 400, the second was $(8, 12)$ with 91 pairs, the third was $(9, 12)$ with 30 pairs, the fourth was $(10, 12)$ with 29 pairs, and the fifth was $(9, 13)$ with 26 pairs. The total frequency of dominant strategy equilibria was 242. The total frequency of Nash equilibria was 370. All Nash equilibria were good.

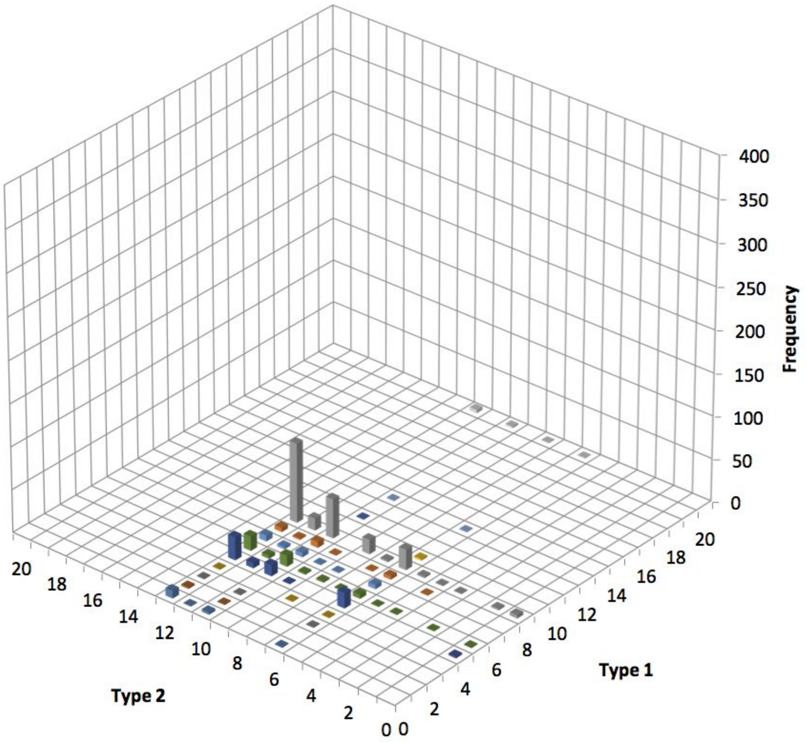


Figure 3: All pairs choices in Treatment PC.

The maximum frequency strategy chosen by Type 1 subjects was $s_1 = 8$ with 224 choices out of 400, the second was $s_1 = 9$ with 72 choices, and the third was $s_1 = 10$ with 70 choices. The total frequency of dominant strategies selected by Type 1 subjects was 296. The maximum frequency strategy chosen by Type 2 was $s_2 = 12$ with 167 choices out of 400, the second was $s_2 = 13$ with 160 choices, and the third was $s_1 = 20$ with 15 choices. Hence, the total frequency of dominant strategies selected by Type 2 subjects was 327.

Figure 4 displays the rates that each type of subjects chose dominant strategies separately as well as the rates that pairs of subjects played dominant strategy equilibria for all periods.

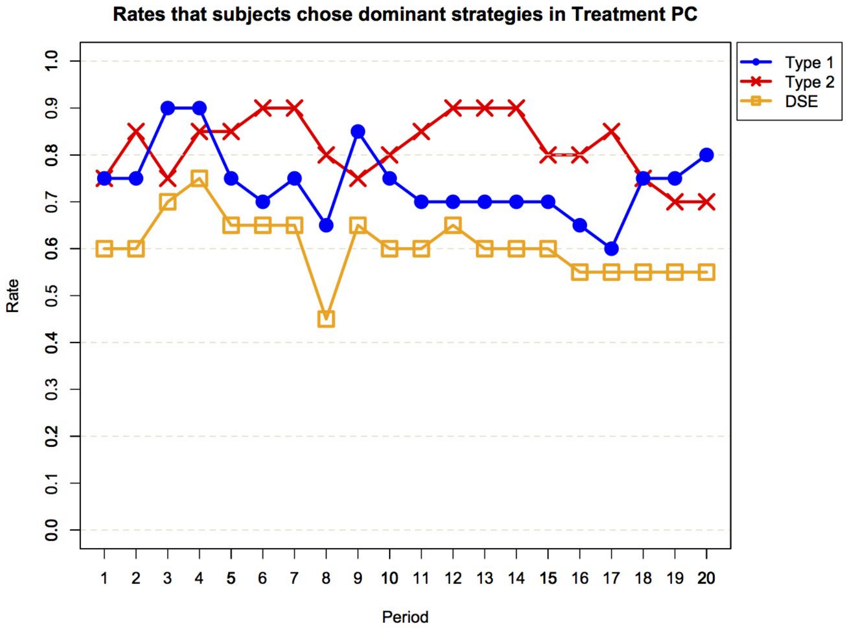


Figure 4: Rates that subjects chose dominant strategies in Treatment PC.

These results are summarized by the following observations.

Observation 2.

(i) *The frequency of dominant strategy equilibria was 61% (= 242/400) across all periods in Treatment PC.*

(ii) *The frequency of good Nash equilibria was 3% (= 370/400) across all periods in Treatment PC.*

(iii) *The frequency that Type 1 subjects chose dominant strategies was 74%(= 296/400) across all periods in Treatment PC.*

(iv) *The frequency that Type 2 subjects chose dominant strategies was 82%(= 327/400) across all periods in Treatment PC.*

5.1.3 Comparing the two mechanisms

We compare the performances of the voluntary contribution mechanism and those of the pivotal mechanism based on the following four indices: (i) the *Pareto efficiency* *index*; (ii) the *individual rationality index*; (iii) the *surplus index*; and (iv) the *decision efficiency index*.

Pareto efficiency index We define the *Pareto efficiency index* at each period as the ratio of Pareto efficient outcomes at that period. The sum of two subjects' payoffs in one pair is equal to 340 at any Pareto efficient outcome. Figure 5 displays the Pareto efficiency indices at each period in Treatments VC and PC. By conducting Fisher's exact tests period by period, we observed that the Pareto efficiency indices for Treatment VC were significantly higher than the Pareto efficiency indices for Treatment PC in 18 out of 20 periods (periods 3–20 at the 1% significance level).

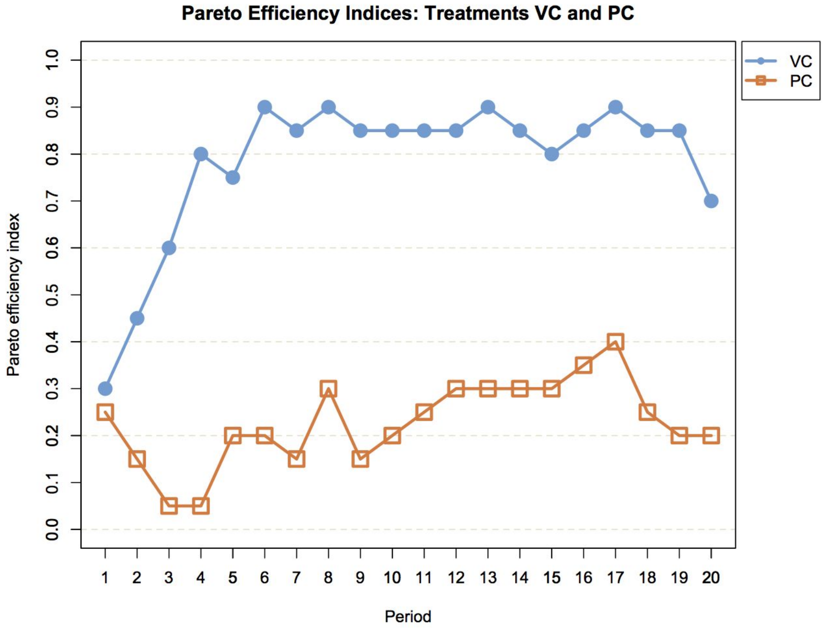


Figure 5: Pareto efficiency index.

Individual rationality index We define the *individual rationality index* at each period as the ratio of individually rational outcomes at that period such that each subject receives a payoff at least as large as their own payoff before participating in the mechanism (160), that is, the payoff the subject receives from their initial endowment of the private good with no public project. Figure 6 displays the individual rationality indices at each period in Treatments VC and PC. According to Fisher's exact tests conducted period by period, the individual rationality indices for Treatment VC were significantly higher than the individual rationality indices for Treatment PC at all periods (at the 1% significance level).

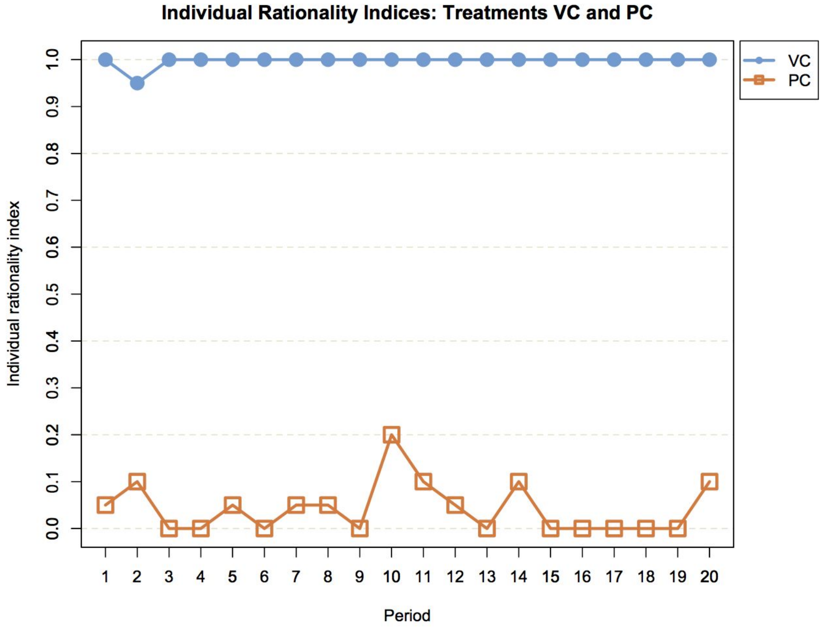


Figure 6: Individual rationality index.

Surplus index We define the *surplus index* by

$$\text{Surplus Index} = \frac{\text{average of sums of two subjects' payoffs} - \text{reference payoff}}{\text{Pareto efficient payoff} - \text{reference payoff}},$$

where the *Pareto efficient payoff* is the sum of two subjects' payoffs at any Pareto efficient outcome, which equals 340, and the *reference payoff* is the sum of two subjects' payoffs that they receive before participating in the mechanism, which equals 320. This index measures how much gain subjects obtain by participating in a mechanism. Figure 7 displays the surplus indices at each period in Treatments VC and PC. By conducting the Mann-Whitney tests period by period, we found that the surplus indices for Treatment VC were significantly higher than the surplus indices for Treatment PC in 18 out of 20 periods (periods 3–20 at the 1% significance level).

Decision efficiency index We define the *decision efficiency index* at each period as the ratio of outcomes at which the public project is undertaken at that period.

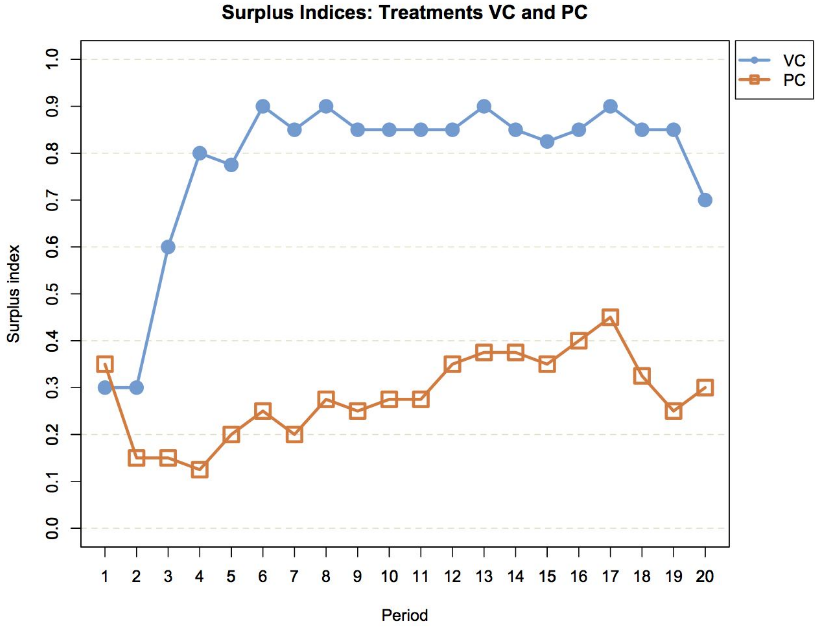


Figure 7: Surplus index.

Figure 8 displays the decision efficiency indices at each period in Treatments VC and PC. According to Fisher's exact test conducted period by period, there was no significant difference in the decision efficiency index between Treatment PC and Treatment VC in 19 out of 20 periods (periods 2–20 at the 5% significance level).

These results are summarized by the following observations.

Observation 3.

(i) *The Pareto efficiency index for Treatment VC was significantly higher than the Pareto efficiency index for Treatment PC.*

(ii) *The individual rationality index for Treatment VC was significantly higher than the individual rationality index for Treatment PC.*

(iii) *The surplus index for Treatment VC was significantly higher than the surplus index for Treatment PC.*

(iv) *There was no significant difference in the decision efficiency index between Treatment VC and Treatment PC.*

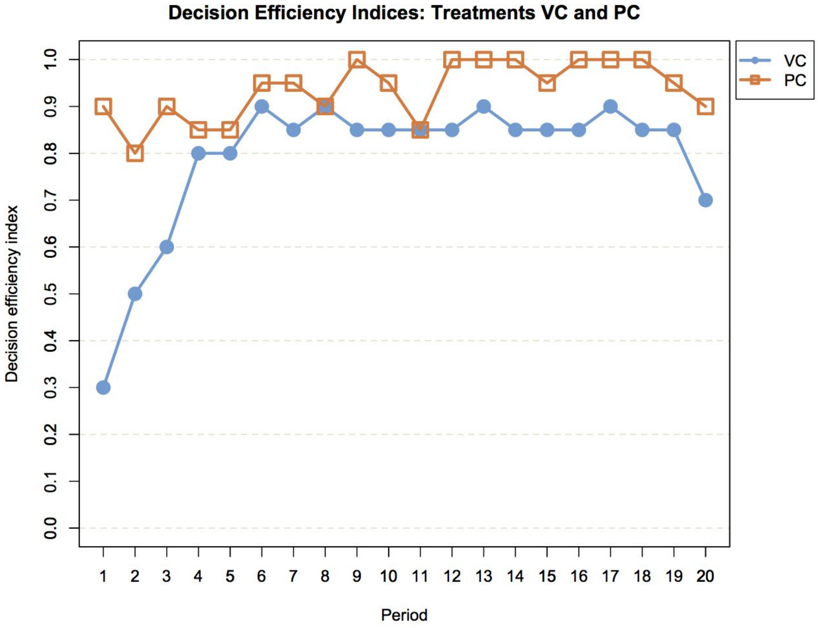


Figure 8: Decision efficiency index.

5.2 Incomplete information

5.2.1 Treatment VI

Figure 9 shows the frequency of distribution of all data in Treatment VI. The maximum frequency pair was $(8, 12)$ with 93 pairs out of 400 outcomes, the second was $(8, 10)$ with 46 pairs, the third was $(4, 12)$ with 27 pairs, and the fourth was $(8, 6)$ with 24 pairs. The total frequency of strategy profiles that achieved decision efficiency was 103.

The maximum frequency strategy chosen by Type 1 subjects was $s_1 = 8$ with 206 choices out of 400, the second was $s_1 = 4$ with 67 choices, and the third was $s_1 = 5$ with 48 choices. The maximum frequency strategy chosen by Type 2 was $s_2 = 12$ with 166 choices out of 400, the second was $s_1 = 10$ with 93 choices, and the third was $s_1 = 6$ with 63 choices.

Figure 10 demonstrates the rates that Type 1 and Type 2 subjects chose $s_1 = 8$ and $s_2 = 12$, respectively. In the first 3 periods, no more than 20% of subjects chose $s_1 = 8$ or $s_2 = 12$. As the periods advanced, the rate of Type 1 subjects who chose $s_1 = 8$ and the rate of Type 2 subjects who chose $s_2 = 12$ increased, but they never exceeded 80%. By conducting the Wilcoxon signed rank tests period by period,

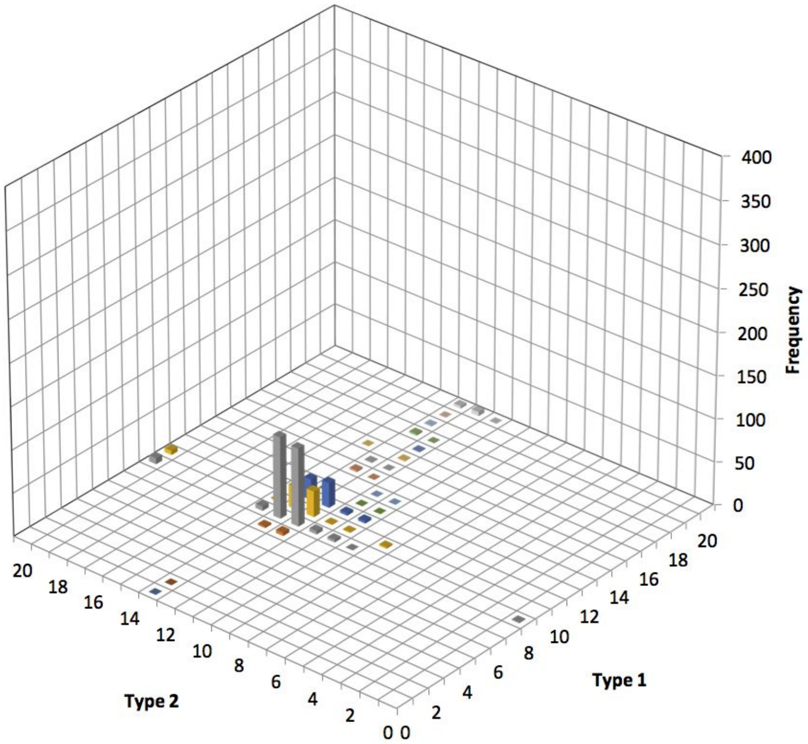


Figure 9: All pairs choices in Treatment VI.

we found that Type 1 subjects' median choice was significantly different from 8 in 14 out of 20 periods (periods 1–3, 5–6, and 8–10 at the 1% significance level; and periods 4, 7, 11, and 13–15 at the 5% significant level) while in the final 5 periods, there were no significant differences. Additionally, we found that Type 2 subjects' median choice was not significantly different from 12 in all periods (periods 1–14 and 18–20 at the 1% significance level; and periods 15–17 at the 5% significance level).

These results are summarized by the following observations.

Observation 4.

(i) *The frequency of the strategy profile (8,12) was 23%(= 93/400) across all periods in Treatment VI.*

(ii) *The frequency that Type 1 subjects chose* $s_1 = 8$ *was 52%(= 206/400) across all periods in Treatment VI.*

(iii) *The frequency that Type 2 subjects chose* $s_2 = 12$ *was 42%(= 166/400) across all periods in Treatment VI.*

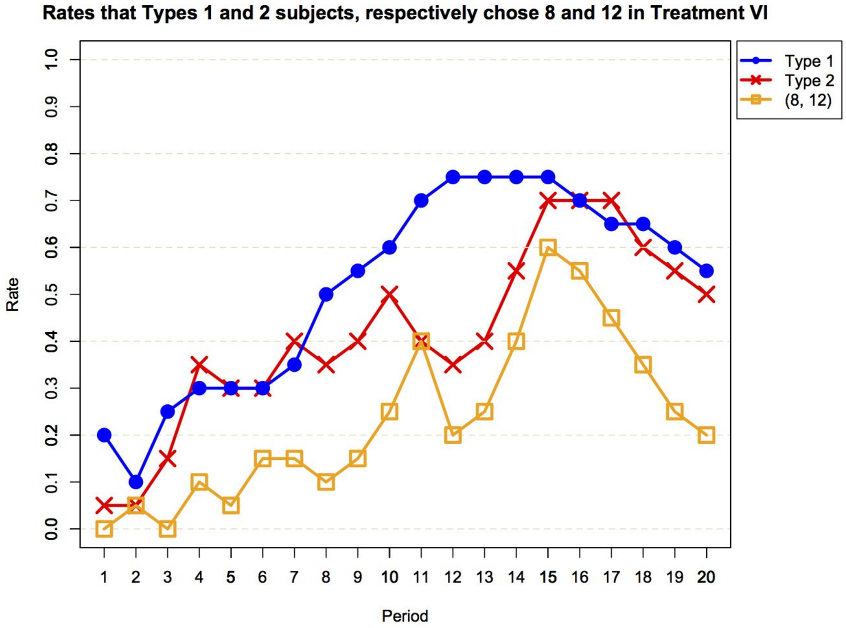


Figure 10: Rates that Types 1 and 2, respectively, subjects chose 8 and 12 in Treatment VI.

5.2.2 Treatment PI

Figure 11 shows the frequency of distribution of all data in Treatment PI. The maximum frequency pair was $(8, 12)$ with 97 pairs out of 400, the second was $(8, 13)$ with 85 pairs, the third was $(9, 12)$ with 57 pairs, and the fourth was $(9, 13)$ with 56 pairs. The total frequency of dominant strategy equilibria was 295. The total frequency of strategy profiles corresponding to Nash equilibria was 371. The total frequency of strategy profiles corresponding to bad Nash equilibria was 0.

The maximum frequency strategy chosen by Type 1 was $s_1 = 8$ with 213 choices out of 400, the second was $s_1 = 9$ with 128 choices, and the third was $s_1 = 20$ with 9 choices. The total frequency of dominant strategies was 341. The maximum frequency strategy chosen by Type 2 was $s_2 = 12$ with 180 choices out of 400, the second was $s_2 = 13$ with 166 choices, and the third was $s_1 = 11$ with 18 choices. Hence, the total frequency of dominant strategies was 346.

Figure 12 displays the rates that subjects chose dominant strategies separately as well as the rates that pairs of subjects played dominant strategy equilibria for all periods. The rates that subjects chose dominant strategies were between 75% and 95% during all periods.

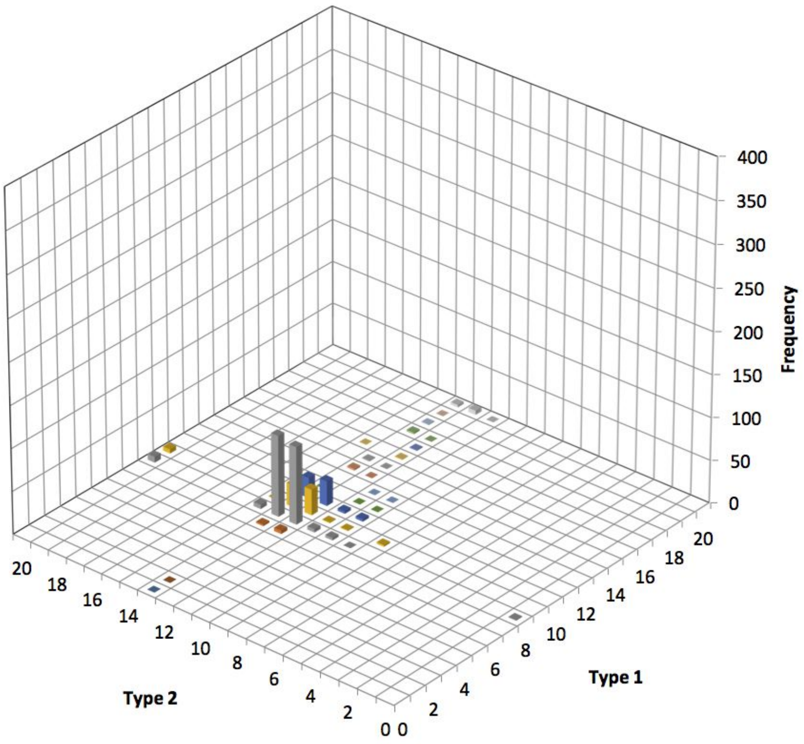


Figure 11: All pairs choices in Treatment PI.

These results are summarized by the following observations.

Observation 5.

(i) *The frequency of dominant strategy equilibria was 74% (= 295/400) across all periods in Treatment PI.*

(ii) *The frequency of strategy profiles corresponding to good Nash equilibria in Treatment PC was 93%(= 371/400) across all periods in Treatment PI.*

(iii) *The frequency that Type 1 subjects chose dominant strategies was 85%(= 341/400) across all periods in Treatment PI.*

(iv) *The frequency that Type 2 subjects chose dominant strategies was 87%(= 346/400) across all periods in Treatment PI.*

5.2.3 Comparing the two mechanisms

As is the case in the setting of complete information, we compare the performances of the voluntary contribution mechanism and those of the pivotal mechanism based

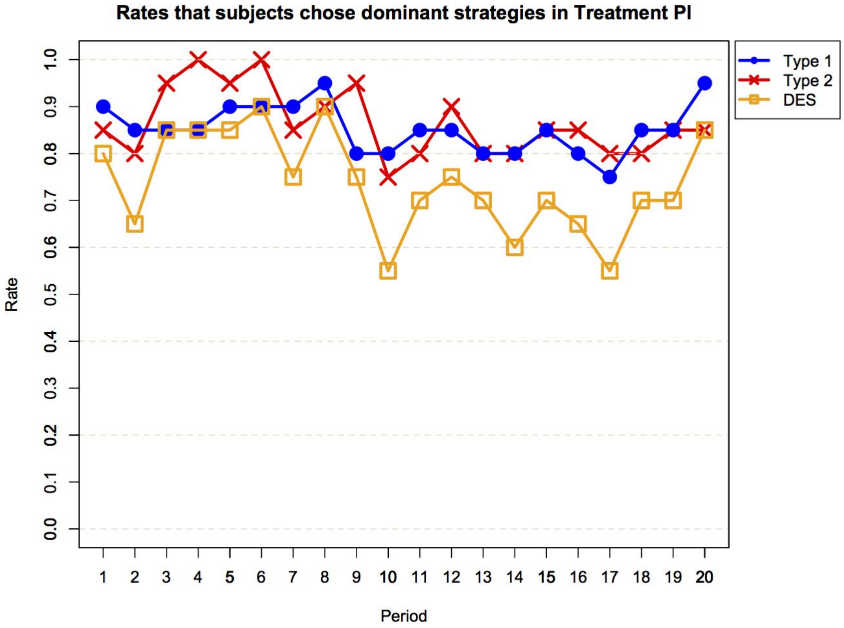


Figure 12: Rates that subjects chose dominant strategies in Treatment PI.

on the above four indices.

Pareto efficiency index Figure 13 displays the Pareto efficiency indices at each period in Treatments VI and PI. By conducting Fisher's exact tests period by period, we found that the Pareto efficiency indices for Treatment VI were not significantly different from the Pareto efficiency indices for Treatment PI in 18 out of 20 periods (periods 1–14 and 17–20 at the 5% significance level).

Individual rationality index Figure 14 displays the individual rationality index at each period in Treatments VI an PI. By conducting Fisher's exact tests period by period, we found that the individual rationality indices for Treatment VI were significantly higher than the individual rationality indices for Treatment PI in all periods (at the 1% significance level).

Surplus index Figure 15 displays the surplus indices at all periods in Treatments VI and PI. By conducting the Mann-Whitney tests period by period, we found that no significant difference in the surplus index between Treatment VI and Treatment PI in 15 out of 20 periods (periods 2, 6–8, and 10–20 at the 5% signifi-

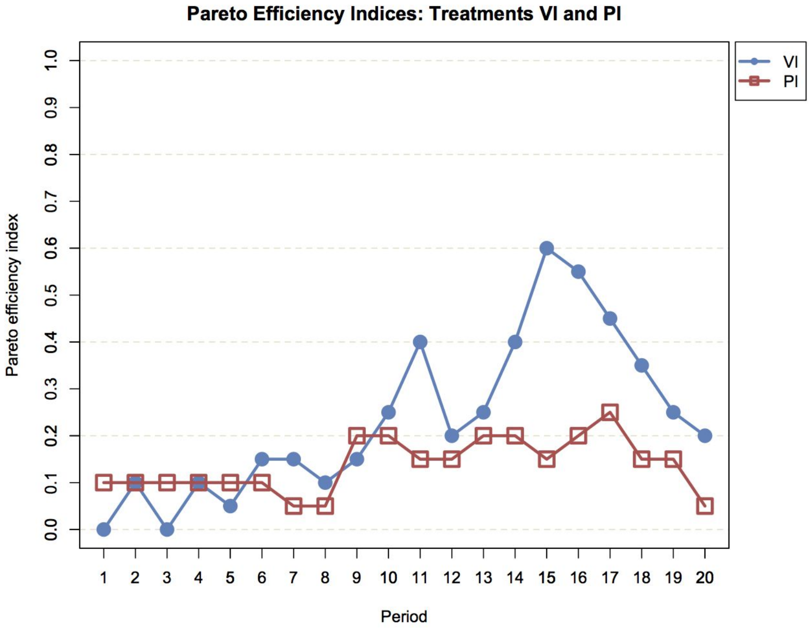


Figure 13: Pareto efficiency index.

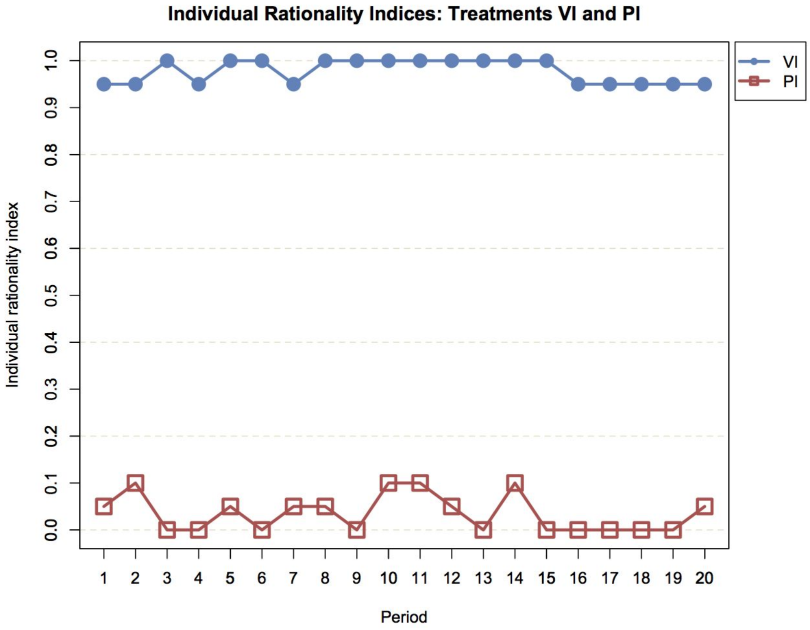


Figure 14: Individual rationality index.

cance level).

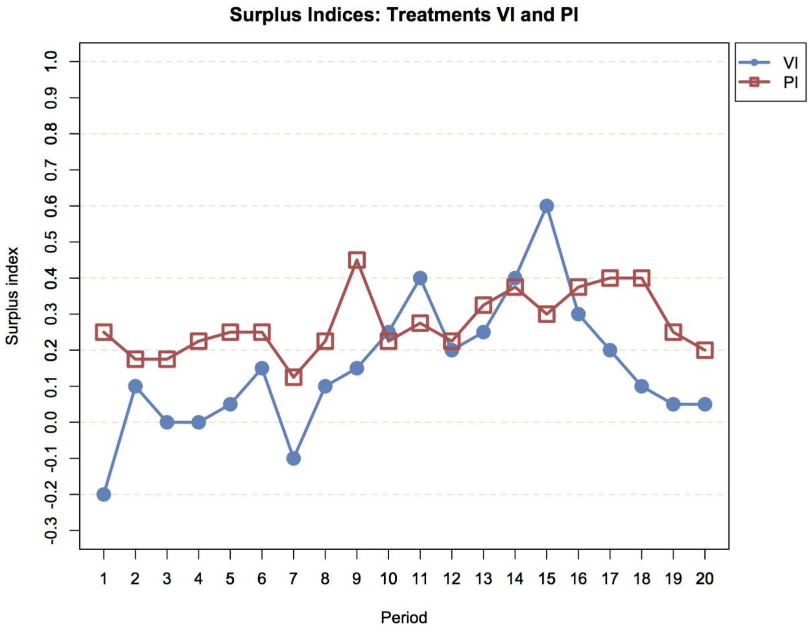


Figure 15: Surplus index.

Decision efficiency index Figure 16 displays the decision efficiency indices at all periods in Treatments VI and PI. According to Fisher's exact tests conducted period by period, the decision efficiency indices for Treatment PI were significantly higher than the decision efficiency indices for Treatment VI in all 20 periods (periods 1–14 and 16–20 at the 1% significance level; and period 15 at the 5% significance level).

These results are summarized by the following observations.

Observation 6.

(i) *There was no significant difference in the Pareto efficiency index between Treatment VI and Treatment PI.*

(ii) *The individual rationality index for Treatment VI was significantly higher than the individual rationality index for Treatment PI.*

(iii) *There was no significant difference in the surplus index between Treatment VI and Treatment PI.*

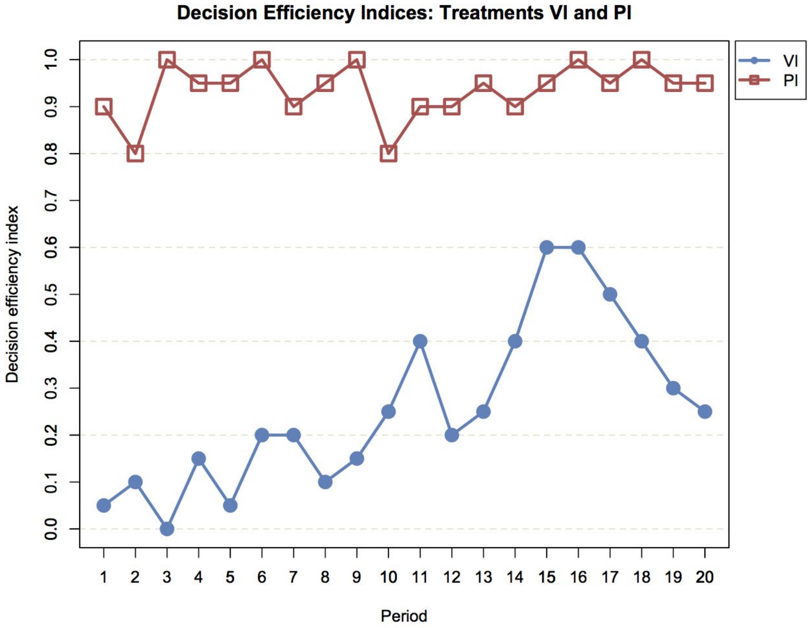


Figure 16: Decision efficiency index.

(iv) *The decision efficiency index for Treatment VI was significantly lower than the decision efficiency index for Treatment PI.*

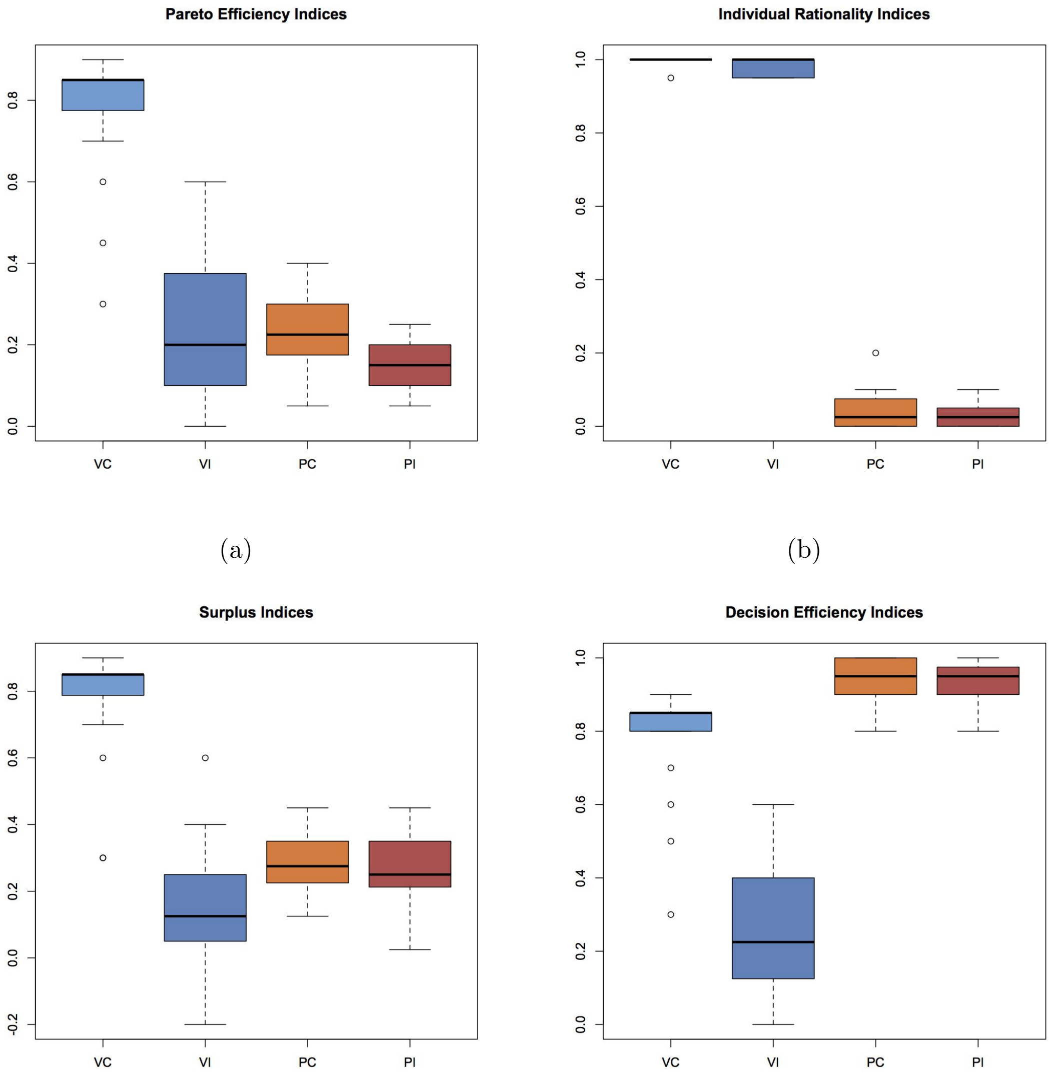
5.3 Informational effects

5.3.1 The voluntary contribution mechanism

We investigate whether a setting where each subject knows the other subject's payoff affects the performance of the voluntary contribution mechanism by comparing Treatment VC with Treatment VI.

By conducting Fisher's exact tests period by period, we found that the frequency of pairs who chose $(8, 12)$ in Treatment VI was significantly lower than the frequency of pairs who chose $(8, 12)$ in Treatment VC in 19 out of 20 periods (periods 1 and 16 at the 5% significance level; and periods 2–14 and 17–20 at the 1% significance level).

We also compare Treatment VC with Treatment VI based on the above four indices. Figure 17(a) is the box plot that displays the distribution of Pareto efficiency indices for Treatments VC and VI. By conducting Fisher's exact tests period by period, we found that the Pareto efficiency indices for Treatment VI were significantly



(a) (b)

(c) (d)

Figure 17: Box plot for each index. (a) Pareto efficiency index. (b) Individual rationality index. (c) Surplus index. (d) Decision efficiency index.

lower than the Pareto efficiency indices for Treatment VC in all 20 periods (at the 1% significance level).

Figure 17(b) is the box plot that displays the distribution of individual rationality indices for Treatments VC and VI. By conducting Fisher's exact tests period by period, we found that there was no significant difference in the individual rationality index between Treatment VC and Treatment VI in all 20 periods (at the 5% significance level).

Figure 17(c) is the box plot that displays the distribution of surplus indices for Treatments VC and VI. By conducting the Mann-Whitney tests period by period, we found that the surplus indices for Treatment VI were significantly lower than the surplus indices for Treatment VC in 19 out of 20 periods (periods 16 at the 5% significance level; and periods 1–14 and 17–20 at the 1% significance level).

Figure 17(d) is the box plot that displays the distribution of decision efficiency indices for Treatments VC and VI. By conducting Fisher's exact tests period by period, we found that the decision efficiency indices for Treatment VI were significantly lower than the decision efficiency indices for Treatment VC in 17 out of 20 periods (periods 2, 17, and 20 at the 5% significance level; and periods 3–14, 18, and 19 at the 1% significance level).

These results are summarized by the following observations.

Observation 7.

(i) *The frequency of the strategy profile* $(8, 12)$ *corresponding the unique twice iteratively undominated Nash equilibrium in Treatment VC was significantly higher than that in Treatment VI.*

(ii) *The Pareto efficiency index for Treatment VC was significantly higher than the Pareto efficiency index for Treatment VI.*

(iii) *There was no significant difference in the individual rationality index between Treatment VC and Treatment VI.*

(iv) *The surplus index for Treatment VC was significantly higher than the surplus index for Treatment VI.*

(v) *The decision efficiency index for Treatment VC was significantly higher than the decision efficiency index for Treatment VI.*

5.3.2 The pivotal mechanism

We also investigate whether a setting where each subject knows the other subject's payoff affects the performance of the pivotal mechanism by comparing Treatment PC with Treatment PI.

According to Fisher's exact tests conducted period by period, there was no significant difference in the frequency of dominant strategy equilibria between Treatment PC and Treatment PI in 18 out of 20 periods (periods 1–7 and 9–19 at the 5% significance level).

We also compare Treatment PC with Treatment PI based on the above four indices. Figure 17(a) is the box plot that displays the distribution of Pareto efficiency indices for Treatments PC and PI. By conducting Fisher's exact tests period by period, we found that there was no significant difference in the Pareto efficiency index between Treatment PC and Treatment PI in 19 out of 20 periods (periods 1–16 and 18–20 at the 5% significance level).

Figure 17(b) is the box plot that displays the distribution of individual rationality indices for Treatments PC and PI. By conducting Fisher's exact tests period by period, we found that there was no significant difference in the individual rationality index between Treatment PC and Treatment PI in all 20 periods (at the 5% significance level).

Figure 17(c) is the box plot that displays the distribution of surplus indices for Treatments PC and PI. By conducting the Mann-Whitney tests period by period, we found that the there was no significant difference in the surplus index between Treatment PC and Treatment PI in all 20 periods (at the 5% significance level).

Figure 17(d) is the box plot that displays the distribution of decision efficiency indices for Treatments PC and PI. By conducting Fisher's exact tests period by period, we found that there was no significant difference in decision efficiency index between Treatment PC and Treatment PI in all 20 periods (at the 5% significance level).

These results are summarized by the following observations.

Observation 8.

(i) *There was no significant difference in the frequency of dominant strategy equilibria between Treatment PC and Treatment PI.*

(ii) *There was no significant difference in the Pareto efficiency index between Treatment PC and Treatment PI.*

(iii) *There was no significant difference in the individual rationality index between Treatment PC and Treatment PI.*

(iv) *There was no significant difference in the surplus index between Treatment PC and Treatment PI.*

(v) *There was no significant difference in the decision efficiency index between Treatment PC and Treatment PI.*

6 Concluding remarks

In our experiment, we used the payoff tables to simplify the presentation of the two mechanisms for subjects. We provided no explanation regarding the rules of the mechanisms or how the payoff tables were constructed. The rule of the pivotal mechanism is more complicated than that of the voluntary contribution mechanism. Using payoff tables only allows for a comparison of the two mechanisms holding their degree of transparency constant. Although payoff tables are somewhat unrealistic for potential applications of these mechanisms in the field, we selected maximally-transparent conditions as a first step in this initial experiment.

We observed that the voluntary contribution mechanism worked better than the pivotal mechanism under complete information in which each subject knew all subjects' payoffs. Since the rule of the pivotal mechanism is more difficult to understand, it is natural to expect that the voluntary contribution mechanism would perform better than the pivotal mechanism even when the rules of the mechanisms are explained under complete information.

On the other hand, under incomplete information in which each subject knew their own payoff but not the payoffs of the other subjects, we cannot conclude which mechanism was better: the pivotal mechanism worked better than the voluntary contribution mechanism from the viewpoint of the decision efficiency index, but the converse was true for the individual rationality index, and there was no significant difference between the two mechanisms in the Pareto efficiency and the surplus indices. Hence, we need to conduct another experiment under incomplete information to check whether the pivotal mechanism performs better than the voluntary contribution mechanism when the rules of the two mechanisms are explained. This is left for future experiments.

Regarding the voluntary contribution mechanism, Marks and Croson (1999) observed little difference on the experimental results between the complete information treatment and the incomplete information treatment. Neither the decision efficiency nor the proportion of Nash equilibrium strategies subjects played was significantly different for the two treatments.[10] This observation is different from ours. There are two main differences in the experimental settings of the Marks-Croson experiment and our experiment, which may cause different observations. First, each group consisted of five subjects in the Marks-Croson experiment, whereas there were two subjects in each group in our experiment. Second, only the rule of the mechanism was explained to the subjects in the Marks-Croson experiment, while only the payoff tables were used in ours. An open question remains when conducting an experiment by providing payoff tables and/or an explanation of the rule to subjects when there are more than two subjects in each group. We leave this issue for the future.

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[10] Marks and Croson (1999) examined another incomplete information treatment in which each subject knew the sum of the public project valuations of the other subjects but not their distribution. There were no significant differences in the experimental results among the three treatments with the exception of the convergence of group contributions toward the equilibrium level over time. In the incomplete information-known sum treatment, no convergence was observed.

A Appendix: Proofs

A.1 Proof of Fact 1

A.1.1 The “if” part

Suppose, by contradiction, that $(y, t) \notin P(v)$. Then, there is $(y', t') \in A$ such that $u_i(y', t'_i; v_i) \geq u_i(y, t_i; v_i)$ for each $i \in \{1, 2\}$ with a strict inequality for some $i \in \{1, 2\}$. There are two cases.

- **Case 1: $\boldsymbol{v_1 + v_2 \geq c}$.** Then,

$$u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2) > u_1(y, t_1; v_1) + u_2(y, t_2; v_2) = v_1 + v_2 - c + \omega_1 + \omega_2.$$

If $y' = 1$, then

$$\begin{aligned} v_1 + v_2 + \omega_1 + \omega_2 + t'_1 + t'_2 &= u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2) \\ &> u_1(y, t_1; v_1) + u_2(y, t_2; v_2) \\ &= v_1 + v_2 - c + \omega_1 + \omega_2, \end{aligned}$$

which implies $-(t'_1 + t'_2) < c$, a contradiction. If $y' = 0$, then

$$\begin{aligned} \omega_1 + \omega_2 + t'_1 + t'_2 &= u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2) \\ &> u_1(y, t_1; v_1) + u_2(y, t_2; v_2) \\ &= v_1 + v_2 - c + \omega_1 + \omega_2, \end{aligned}$$

which implies $-(t'_1 + t'_2) < c - (v_1 + v_2) \leq 0$, a contradiction.

- **Case 2: $\boldsymbol{v_1 + v_2 < c}$.** Then,

$$u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2) > u_1(y, t_1; v_1) + u_2(y, t_2; v_2) = \omega_1 + \omega_2.$$

If $y' = 1$, then

$$\begin{aligned} v_1 + v_2 + \omega_1 + \omega_2 + t'_1 + t'_2 &= u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2) \\ &> u_1(y, t_1; v_1) + u_2(y, t_2; v_2) \\ &= \omega_1 + \omega_2, \end{aligned}$$

which implies $-(t'_1 + t'_2) < v_1 + v_2 < c$, a contradiction. If $y' = 0$, then

$$\begin{aligned}\omega_1 + \omega_2 + t'_1 + t'_2 &= u_1(y', t'_1; v_1) + u_2(y', t'_2; v_2)\\ &> u_1(y, t_1; v_1) + u_2(y, t_2; v_2)\\ &= \omega_1 + \omega_2,\end{aligned}$$

which implies $-(t'_1 + t'_2) < 0$, a contradiction. □

A.1.2 The "only if" part

Let $(y, t) \in P(v)$. There are three cases.

- **Case 1: $\boldsymbol{v_1 + v_2 > c}$.** In this case, it suffices to show that $y = 1$, because this together with $(y, t) \in P(v)$ implies that $u_1(y, t_1; v_1) + u_2(y, t_2; v_2) = v_1 + v_2 - c + \omega_1 + \omega_2$. To show this, suppose by contradiction that $y = 0$. Then, $u_1(y, t_1; v_1) = \omega_1 + t_1$, $u_2(y, t_2; v_2) = \omega_2 + t_2$, and $-(t_1 + t_2) \geq 0$. Without loss of generality, assume $t_1 \geq t_2$. Let $(y', t') \in A$ be such that $y' = 1$, $-(t'_1 + t'_2) = c$, and $t'_1 = t_1 - v_1$. Since $v_1 + v_2 > c$, $v_1 + v_2 - c = v_1 + v_2 + (t'_1 + t'_2) > 0 \geq t_1 + t_2$, which implies that $v_2 + t'_2 > t_2$. It then follows that

$$\begin{aligned}u_1(y', t'_1; v_1) &= v_1 + t'_1 + \omega_1 = t_1 + \omega_1 = u_1(y, t_1; v_1),\\ u_2(y', t'_2; v_2) &= v_2 + t'_2 + \omega_1 > t_2 + \omega_2 = u_2(y, t_2; v_2),\end{aligned}$$

which contradict $(y, t) \in P(v)$.

- **Case 2: $\boldsymbol{v_1 + v_2 < c}$.** In this case, it suffices to show that $y = 0$, because this together with $(y, t) \in P(v)$ implies that $u_1(y, t_1; v_1) + u_2(y, t_2; v_2) = \omega_1 + \omega_2$. To show this, suppose by contradiction that $y = 1$. Then, $u_1(y, t_1; v_1) = v_1 + \omega_1 + t_1$, $u_2(y, t_2; v_2) = v_2 + \omega_2 + t_2$, and $-(t_1 + t_2) \geq c > 0$. Without loss of generality, assume $v_1 + t_1 \geq v_2 + t_2$. Let $(y', t') \in A$ be such that $y' = 0$, $t'_1 + t'_2 = 0$, and $t'_1 = v_1 + t_1$. Since $v_1 + v_2 < c$, $t'_1 + t'_2 = 0 > v_1 + v_2 - c \geq v_1 + v_2 + (t_1 + t_2)$, which implies that $t'_2 > v_2 + t_2$. It then follows that

$$\begin{aligned}u_1(y', t'_1; v_1) &= \omega_1 + t'_1 = v_1 + t_1 + \omega_1 = u_1(y, t_1; v_1),\\ u_2(y', t'_2; v_2) &= \omega_1 + t'_2 > v_2 + t_2 + \omega_2 = u_2(y, t_2; v_2),\end{aligned}$$

which contradict $(y, t) \in P(v)$.

- **Case 3: $\boldsymbol{v_1 + v_2 = c}$.** Then, either $y = 1$ or $y = 0$. If $y = 1$, then it follows from $(y,t) \in P(v)$ that $u_1(y,t_1;v_1) + u_2(y,t_2;v_2) = v_1 + v_2 - c + \omega_1 + \omega_2$. If $y = 0$, then it follows from $(y,t) \in P(v)$ that $v_1 + v_2 = c$, that $u_1(y,t_1;v_1) + u_2(y,t_2;v_2) = \omega_1 + \omega_2 = v_1 + v_2 - c + \omega_1 + \omega_2$. $\square$

A.2 Proof of Fact 2

We prove this fact by contraposition. Suppose that an allocation $(y,t) \in A$ is not decision efficient for v. Then, $y \notin \arg\max_{y' \in \{0,1\}} (v_1 + v_2) \cdot y' - c \cdot y'$. There are two cases.

- **Case 1: $\boldsymbol{y = 1}$.** Then,

$$0 > v_1 + v_2 - c. \tag{2}$$

Let $(y',t') \in A$ be such that $y' = 0$ and for each $i \in \{1,2\}$,

$$t'_i \equiv t_i + v_i + \frac{c - (v_1 + v_2)}{2}.$$

Note that by $-(t_1 + t_2) \geq c$,

$$\begin{aligned} t'_1 + t'_2 &= t_1 + t_2 + v_1 + v_2 + c - (v_1 + v_2) \\ &= t_1 + t_2 + c \\ &\leq 0. \end{aligned}$$

By (2), we have

$$\begin{aligned} u_1(y',t'_1;v_1) &= \omega_1 + t'_1 = \omega_1 + t_1 + v_1 + \frac{c - (v_1 + v_2)}{2} > \omega_1 + t_1 + v_1 = u_1(y,t_1;v_1), \\ u_2(y',t'_2;v_2) &= \omega_2 + t'_2 = \omega_2 + t_2 + v_2 + \frac{c - (v_1 + v_2)}{2} > \omega_2 + t_2 + v_2 = u_2(y,t_2;v_2). \end{aligned}$$

These imply that $(y,t) \notin P(v)$.

- **Case 2: $\boldsymbol{y = 0}$.** Then,

$$v_1 + v_2 - c > 0. \tag{3}$$

Let $(y',t') \in A$ be such that $y' = 1$ and for each $i \in \{1,2\}$,

$$t'_i \equiv t_i - v_i + \frac{v_1 + v_2 - c}{2}.$$

Note that by $-(t_1 + t_2) \geq 0$,

$$\begin{aligned} t'_1 + t'_2 &= t_1 + t_2 - v_1 - v_2 + v_1 + v_2 - c \\ &= t_1 + t_2 - c \\ &\leq -c. \end{aligned}$$

By (3), we have

$$\begin{aligned} u_1(y', t'_1; v_1) &= v_1 + \omega_1 + t'_1 = \omega_1 + t_1 + \frac{v_1 + v_2 - c}{2} > \omega_1 + t_1 = u_1(y, t_1; v_1), \\ u_2(y', t'_2; v_2) &= v_2 + \omega_2 + t'_2 = \omega_2 + t_2 + \frac{v_1 + v_2 - c}{2} > \omega_2 + t_2 = u_2(y, t_2; v_2). \end{aligned}$$

These imply that $(y, t) \notin P(v)$. □

A.3 Proof of Fact 3

A.3.1 The "if" part

Suppose, by contradiction, that $(y, t) \in A$ is not individually rational for v. Then, there exist $i \in \{1, 2\}$ and $(y', t'_i) \in A_i$ such that $u_i(y', t'_i; v_i) > u_i(y, t_i; v_i)$. There are two cases.

- **Case 1: $\boldsymbol{v_i \geq c}$.** Then, $u_i(y', t'_i; v_i) > u_i(y, t_i; v_i) \geq v_i - c + \omega_i$. If $y' = 1$, then $v_i + \omega_i + t'_i > v_i - c + \omega_i$, which implies $-t'_i < c$, a contradiction. If $y' = 0$, then $\omega_i + t'_i > v_i - c + \omega_i$, which implies $-t'_i < c - v_i < 0$, a contradiction.

- **Case 2: $\boldsymbol{v_i < c}$.** Then, $u_i(y', t'_i; v_i) > u_i(y, t_i; v_i) \geq \omega_i$. If $y' = 1$, then $v_i + \omega_i + t'_i > \omega_i$, which implies $-t'_i < v_i < c$, a contradiction. If $y' = 0$, then $\omega_i + t'_i > \omega_i$, which implies $-t'_i < 0$, a contradiction. □

A.3.2 The "only if" part

We prove this by contraposition. Suppose that there are $(y, t) \in A$ and $i \in \{1, 2\}$ such that (1) does not hold. If $v_i \geq c$, then $u_i(y, t_i; v_i) < v_i - c + \omega_i$. Let $(y', t'_i) \in A_i$ be such that $y' = 1$ and $t'_i = -c$. Then, $u_i(y, t_i; v_i) < u_i(y', t'_i; v_i)$, which implies $(y, t) \notin I(v)$. If $v_i < c$, then $u_i(y, t_i; v_i) < \omega_i$. Let $(y', t'_i) \in A_i$ be such that $y' = 0$ and $t'_i = 0$. Then, $u_i(y, t_i; v_i) < u_i(y', t'_i; v_i)$, which implies $(y, t) \notin I(v)$. □

A.4 Proof of Proposition 1

Let $v \in V$. We proceed in two steps.[11]

Step 1: For each $\boldsymbol{i \in \{1,2\}}$, $\boldsymbol{U(S_i^{\mathrm{VC}}; (\Gamma^{\mathrm{VC}}, v)) = [0, v_i[}$. Let $i \in \{1,2\}$. Without loss of generality, assume $i = 1$.

○ **Substep 1-1: Any $\boldsymbol{s_1 \in [v_1, \omega_1]}$ is weakly dominated.** Let $s_1 \in [0, v_1[$. Since $\omega_2 \geq c$ by A1, there exists $s_2^* \in S_2^{\mathrm{VC}}$ such that $s_1 + s_2^* = c$. Let $s_2^{**} \in S_2^{\mathrm{VC}}$ be such that $s_1' + s_2^{**} = c$. Note that $s_2^{**} < s_2^*$. Let $s_2' \in S_2^{\mathrm{VC}}$. There are three cases.

- **Case 1: $\boldsymbol{s_2' \in [0, s_2^{**}[}$.** Then, $g^{\mathrm{VC}}(s_1, s_2') = g^{\mathrm{VC}}(s_1', s_2') = (0, (0,0))$, which implies

$$u_1(g_1^{\mathrm{VC}}(s_2, s_2'); v_1) = \omega_1 = u_1(g_1^{\mathrm{VC}}(s_2', s_2'); v_1).$$

- **Case 2: $\boldsymbol{s_2' \in [s_2^{**}, s_2^*[}$.** Then, $g^{\mathrm{VC}}(s_1, s_2') = (0, (0,0))$ and $g^{\mathrm{VC}}(s_1', s_2') = (1, (-s_1', -s_2'))$. Since $s_1' \geq v_1$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2'); v_1) = \omega_1 \geq v_1 + \omega_1 - s_1' = u_1(g_1^{\mathrm{VC}}(s_1', s_2'); v_1).$$

- **Case 3: $\boldsymbol{s_2' \in [s_2^*, \omega_2]}$.** Then, $g^{\mathrm{VC}}(s_1, s_2') = (1, (-s_1, -s_2'))$ and $g^{\mathrm{VC}}(s_1', s_2') = (1, (-s_1', -s_2'))$. Since $s_1' > s_1$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2'); v_1) = v_1 + \omega_1 - s_1 > v_1 + \omega_1 - s_1' = u_1(g_1^{\mathrm{VC}}(s_1', s_2'); v_1).$$

By Cases 1–3, we can conclude that s_1' is weakly dominated.

○ **Substep 1-2: Any $\boldsymbol{s_1 \in [0, v_1[}$ is not weakly dominated.** Note that there exists $s_2^* \in S_2^{\mathrm{VC}}$ be such that $s_1 + s_2^* = c$. Let $s_1' \in S_1^{\mathrm{VC}} \setminus \{s_1\}$. There are two cases.

- **Case 1: $\boldsymbol{s_1 > s_1'}$.** Then $g^{\mathrm{VC}}(s_1, s_2^*) = (1, (-s_1, -s_2^*))$ and $g^{\mathrm{VC}}(s_1', s_2^*) = (0, (0,0))$. Since $v_1 > s_1$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2^*); v_1) = v_1 + \omega_1 - s_1 > \omega_1 = u_1(g_1^{\mathrm{VC}}(s_1', s_2^*); v_1).$$

- **Case 2: $\boldsymbol{s_1 < s_1'}$.** Then, $g^{\mathrm{VC}}(s_1, s_2^*) = (1, (-s_1, -s_2^*))$ and $g^{\mathrm{VC}}(s_1', s_2^*) =$

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[11] Let $a, b \in \mathbb{R}$ be such that $a \leq b$. Then, we denote by $[a, b]$ and $]a, b[$ the closed interval from a to b and the open interval from a to b, respectively. We also denote by $[a, b[$ and $]a, b]$ the half-open intervals from a to b.

$(1, (-s'_1, -s^*_2))$. Since $s'_1 > s_1$,

$$u_1(g^{\text{VC}}_1(s_1, s^*_2); v_1) = v_1 + \omega_1 - s_1 > v_1 + \omega_1 - s'_1 = u_1(g^{\text{VC}}_1(s'_1, s^*_2); v_1).$$

By Cases 1–2, s_1 is not weakly dominated.

Step 2: $\boldsymbol{\mathcal{A}(\Gamma^{\text{VC}}, v) \subseteq C(v)}$. There are two cases.

- **Case 1: $\boldsymbol{v_1 + v_2 > c}$.** For each $i \in \{1, 2\}$, let

$$\underline{s}_i \equiv \inf \{s_i \in [0, v_i[: \ s_i + s_j = c \ \text{ for some } s_j \in [0, v_j[\} .$$

Note that under A1, for each $s_i \in [0, \underline{s}_i]$ and each $s_j \in [0, v_j[$, $g^{\text{VC}}(s_i, s_j) = (0, (0, 0))$.

○ **Substep 2-1: For each $\boldsymbol{i \in \{1, 2\}}$ and each $\boldsymbol{s'_i \in [0, \underline{s}_i]}$, $\boldsymbol{s'_i}$ is weakly dominated in the game $\boldsymbol{(\Gamma^{\text{VC}}(U), v)}$.** Let $i \in \{1, 2\}$. Without loss of generality, assume $i = 1$. Let $s_1, s'_1 \in [0, v_1[$ be such that $s'_1 \leq \underline{s}_1 < s_1$. Then, there exists $s^*_2 \in [0, v_2[$ such that $s_1 + s^*_2 = c$. It follows that $g^{\text{VC}}(s_1, s_2) = (0, (0, 0))$ if $s_2 < s^*_2$ and $g^{\text{VC}}(s_1, s_2) = (1, (-s_1, -s_2))$ otherwise. Let $s_2 \in [0, v_2[$. We also distinguish two subcases.

1. If $s_2 < s^*_2$, then $g^{\text{VC}}_1(s_1, s_2) = g^{\text{VC}}_1(s'_1, s_2) = (0, (0, 0))$, which implies

$$u_1(g^{\text{VC}}_1(s_1, s_2); v_1) = \omega_1 = u_1(g^{\text{VC}}_1(s'_1, s_2); v_1).$$

2. If $s_2 \geq s^*_2$, then $g^{\text{VC}}_1(s_1, s_2) = (1, (-s_1, -s_2))$ and $g^{\text{VC}}_1(s'_1, s_2) = (0, (0, 0))$. Since $v_1 > s_1$,

$$u_1(g^{\text{VC}}_1(s_1, s_2); v_1) = v_1 + \omega_1 - s_1 > \omega_1 = u_1(g^{\text{VC}}_1(s'_1, s_2); v_1).$$

Hence s'_1 is weakly dominated.

○ **Substep 2-2: For each $\boldsymbol{i \in \{1, 2\}}$ and each $\boldsymbol{s_i \in]\underline{s}_i, v_i[}$, $\boldsymbol{s_i}$ is not weakly dominated in the game $\boldsymbol{(\Gamma^{\text{VC}}(U), v)}$.** Let $i \in \{1, 2\}$. Without loss of generality, assume $i = 1$. Let $s_1 \in]\underline{s}_1, v_1[$. Note that there exists $s^*_2 \in [0, v_2[$ such that $s_1 + s^*_2 = c$. Let $s'_1 \in [0, v_1[\ \setminus \{s_1\}$. We also distinguish two subcases.

1. If $s_1 > s'_1$, then $g^{\text{VC}}_1(s_1, s^*_2) = (1, (-s_1, -s^*_2))$ and $g^{\text{VC}}_1(s'_1, s^*_2) = (0, (0, 0))$. Since $v_1 > s_1$,

$$u_1(g^{\text{VC}}_1(s_1, s^*_2); v_1) = v_1 + \omega_1 - s_1 > \omega_1 = u_1(g^{\text{VC}}_1(s'_1, s^*_2); v_1).$$

2. If $s_1 < s_1'$, then $g_1^{\mathrm{VC}}(s_1, s_2^*) = (1, (-s_1, -s_2^*))$ and $g_1^{\mathrm{VC}}(s_1', s_2^*) = (1, (-s_1', -s_2^*))$. Since $s_1 < s_1'$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2^*); v_1) = v_1 + \omega_1 - s_1 > v_1 + \omega_1 - s_1' = u_1(g_1^{\mathrm{VC}}(s_1', s_2^*); v_1).$$

Hence s_1 is not weakly dominated.

○ **Substep 2-3: $\boldsymbol{\mathcal{E}(\Gamma^{\mathrm{VC}}, v) = \{(s_1, s_2) \in [0, v_1[\times [0, v_2[: s_1 + s_2 = c\}}$.** We first show that $\mathcal{E}(\Gamma^{\mathrm{VC}}, v) \subseteq \{(s_1, s_2) \in [0, v_1[\times [0, v_2[: s_1 + s_2 = c\}$. Let $s \in \mathcal{E}(\Gamma^{\mathrm{VC}}, v)$. Suppose, by contradiction, that $s_1 + s_2 \neq c$. If $s_1 + s_2 > c$, there exists $i \in \{1, 2\}$ with $s_i > 0$. Let $\varepsilon > 0$ be such that $s_i + s_j > c + \varepsilon$. Let $s_i' = s_i - \varepsilon$. Then,

$$u_i(g_i^{\mathrm{VC}}(s_i', s_j); v_i) = v_i + \omega_i - s_i + \varepsilon > v_i + \omega_i - s_i = u_i(g_i^{\mathrm{VC}}(s_i, s_j); v_i),$$

a contradiction. If $s_1 + s_2 < c$, by Substeps 2.1 and 2.2, then $s_1 > \underline{s}_1$ and $s_2 > \underline{s}_2$. Let $i \in \{1, 2\}$. Then, there exists s_i' such that $s_i' + s_j = c$. Since $v_i > s_i$,

$$u_i(g_i^{\mathrm{VC}}(s_i', s_j); v_i) = v_i + \omega_i - s_i > \omega_i = u_i(g_i^{\mathrm{VC}}(s_i, s_j); v_i),$$

a contradiction.

We next show that $\{(s_1, s_2) \in [0, v_1[\times [0, v_2[: s_1 + s_2 = c\} \subseteq \mathcal{E}(\Gamma^{\mathrm{VC}}, v)$. Let $s' = (s_1', s_2') \in \{(s_1, s_2) \in [0, v_1[\times [0, v_2[: s_1 + s_2 = c\}$. Then, for each $i, j \in \{1, 2\}$ and each $s_i''(> \underline{s}_i)$,

$$u_i(g_i^{\mathrm{VC}}(s_i', s_j'); v_i) = v_i + \omega_i - s_i \geq u_i(g_i^{\mathrm{VC}}(s_i'', s_j'); v_i).$$

Moreover, since $s_1' + s_2' = c$, both s_1' and s_2' are not weakly dominated. Thus, $s' \in \mathcal{E}(\Gamma^{\mathrm{VC}}, v)$.

○ **Substep 2-4: Concluding.** It follows from Substep 2-3 that $\mathcal{A}(\Gamma^{\mathrm{VC}}, v) \subseteq C(v)$.

• **Case 2: $\boldsymbol{v_1 + v_2 \leq c}$.** To show $\mathcal{A}(\Gamma^{\mathrm{VC}}, v) \subseteq C(v)$, we first show that $\mathcal{E}(\Gamma^{\mathrm{VC}}, v) = [0, v_1[\times [0, v_2[$. It is obvious that $\mathcal{E}(\Gamma^{\mathrm{VC}}, v) \subseteq [0, v_1[\times [0, v_2[$. Therefore, it suffices to show that $[0, v_1[\times [0, v_2[\subseteq \mathcal{E}(\Gamma^{\mathrm{VC}}, v)$. Let $(s_1, s_2) \in [0, v_1[\times [0, v_2[$. Since $s_1 + s_2 < c$, $g^{\mathrm{VC}}(s) = (0, (0, 0))$. It follows that for each $i \in \{1, 2\}$, s_i is not weakly dominated in the game $(\Gamma^{\mathrm{VC}}(U), v)$ and, moreover, (s_1, s_2) is a Nash equilibrium of the game $(\Gamma^{\mathrm{VC}}(U), v)$. Hence $(s_1, s_2) \in \mathcal{E}(\Gamma^{\mathrm{VC}}, v)$. It then follows that $\mathcal{A}(\Gamma^{\mathrm{VC}}, v) = \{(0, (0, 0))\} \subseteq C(v)$. □

B Appendix: Implementation in iterated elimination of weakly dominated strategies

This section shows that the voluntary contribution mechanism cannot implement any sub-correspondence of the core in iterated elimination of weakly dominated strategies. To see this, suppose that $\omega_1 = \omega_2 = 8$ and $c = 8$.

We now consider the case where $v = (7, 7)$. Let $U_i^1 \equiv U(S_i^{\mathrm{VC}}; (\Gamma^{\mathrm{VC}}, v))$ and $U^1 \equiv U_1^1 \times U_2^1$. From the proof of Proposition 1 (Step 2), we obtain the following facts:

- For each $i \in \{1, 2\}$, $U_i^1 = [0, 7[$;
- For each $i \in \{1, 2\}$ and each $s_i \in [0, 1[$, s_i is weakly dominated in the game $(\Gamma^{\mathrm{VC}}(U^1), v)$; and
- For each $i \in \{1, 2\}$ and each $s_i \in U_i^1 \setminus [0, 1[=]1, 7[$, s_i is not weakly dominated in the game $(\Gamma^{\mathrm{VC}}(U^1), v)$.

Therefore, for each $i \in \{1, 2\}$, $U_1^2 \equiv U(U_i^1; (\Gamma^{\mathrm{VC}}(U^1), v)) =]1, 7[$. We now show that for each $i \in \{1, 2\}$ and each $s_i \in U_i^2$, s_i is not weakly dominated in the game $(\Gamma^{\mathrm{VC}}(U^2), v)$. Let $i \in \{1, 2\}$. Without loss of generality, assume $i = 1$. Let $s_1 \in U_1^2$. Note that there exists $s_2^* \in U_2^2$ such that $s_1 + s_2^* = c = 8$. Let $s_1' \in U_1^2 \setminus \{s_1\}$. There are two cases.

• **Case 1: $\boldsymbol{s_1 > s_1'}$.** Then, $g_1^{\mathrm{VC}}(s_1, s_2^*) = (1, (-s_1, -s_2^*))$ and $g_1^{\mathrm{VC}}(s_1', s_2^*) = (0, (0, 0))$. Since $v_1 = 7 > s_1$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2^*); v_1) = v_1 + \omega_1 - s_1 = 8 + (7 - s_1) > 8 = \omega_1 = u_1(g_1^{\mathrm{VC}}(s_1', s_2^*); v_1).$$

• **Case 2: $\boldsymbol{s_1 < s_1'}$.** Then, $g_1^{\mathrm{VC}}(s_1, s_2^*) = (1, (-s_1, -s_2^*))$ and $g_1^{\mathrm{VC}}(s_1', s_2^*) = (1, (-s_1', -s_2^*))$. Since $s_1 < s_1'$,

$$u_1(g_1^{\mathrm{VC}}(s_1, s_2^*); v_1) = v_1 + \omega_1 - s_1 > v_1 + \omega_1 - s_1' = u_1(g_1^{\mathrm{VC}}(s_1', s_2^*); v_1).$$

From Cases 1–2, we can conclude that s_1 is not weakly dominated. This fact implies that the set of iterated elimination of weakly dominated strategies is $]1, 7[\times]1, 7[$. We now consider the strategy profile $(2, 2)$. Then, $(2, 2) \in]1, 7[\times]1, 7[$ but

$g^{\mathrm{VC}}(2,2) = (0,(0,0)) \notin C(v)$. This implies that the voluntary contribution mechanism fails to implement any sub-correspondence of the core in iterated elimination of weakly dominated strategies.

C Appendix: Experimental instructions (PC and VC)

In this experiment, please remember that you cannot talk or communicate with other subjects. If there is communication, the experiment will be stopped at that point.

First, please confirm that the following items are on your desk. If any of the items are missing, please contact an experimenter.

- Instructions (this set of papers)
- Payoff table for practice
- Record sheet for practice
- Pencil and eraser

C.1 Overview

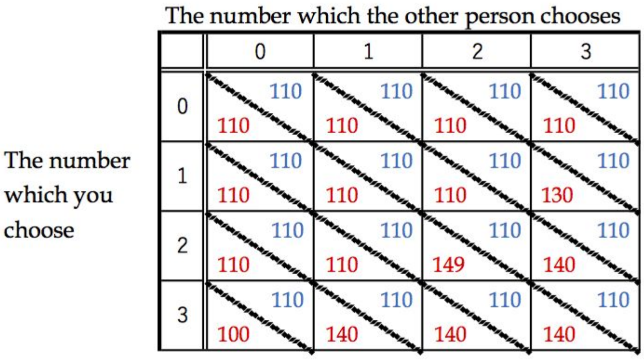
In this experiment, at the beginning of each period, *an experimenter will choose the person you are paired with from the other subjects at random.* The person you are paired with will change each period. This experiment consists of 20 periods. As explained in more detail below, at the beginning of each period, you will make decisions using a computer assigned to you based on the "Payoff table." You will choose one integer number from 1 to 20 in each period. No subject knows who they have been paired with either during or after the experiment.

The rewards you receive after the experiment is complete are proportional to the total payoffs you earn throughout the 20 periods of the experiment. A detailed explanation of the rewards you receive will be provided later in Section C.4 (Rewards).

C.2 Details

You will choose one integer number from 1 to 20 in each period. Your payoff in each period is determined by the "Outcome" of the period. The outcome is determined by the number you choose as well as the number the other person chooses. Your "Payoff table" describes the relation between outcomes and the payoffs you earn.

Your "Payoff table" displays both your payoff and the other person's payoff when the number you choose and the number the other person chooses are determined. The following table is a part of the "Payoff table for practice."



		The number which the other person chooses			
		0	1	2	3
The number which you choose	0	110 110	110 110	110 110	110 110
	1	110 110	110 110	110 110	110 130
	2	110 110	110 110	110 149	110 140
	3	110 100	110 140	110 140	110 140

The vertical line of the above table shows the number you choose, and the horizontal line is the number the other person chooses. The lower left-hand red number is your payoff, and the upper right-hand blue number is the other person's payoff in each cell. The numbers in each cell are displayed in yen.

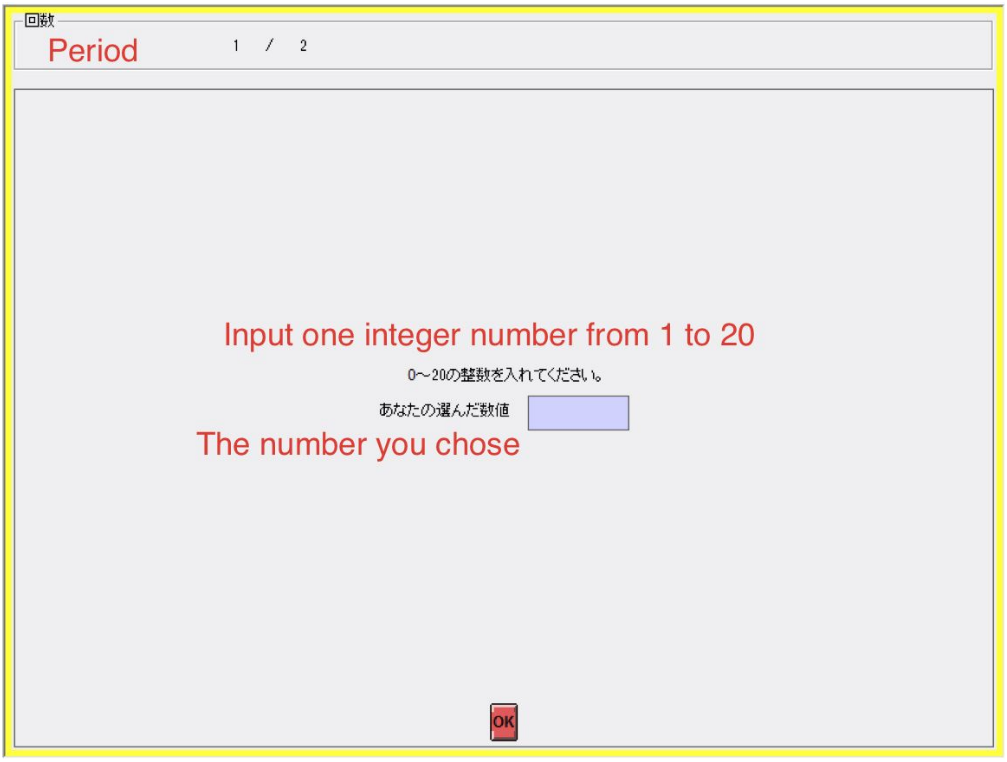
For example, suppose that the number you choose is "3," and the number the other person chooses is "1." In this case, your payoff is "140" when the vertical line is 3 and the horizontal line is 1. Then, the other person's payoff is "100."

The person you are paired with is chosen from the other subjects at random by an experimenter. The person you are paired with has a payoff table in which the vertical line and the horizontal line of your payoff table are reversed.

C.3 Operation

This experiment consists of 20 periods. Here, we will explain how to operate the computer you will use in period 1 of the experiment. The operation after period 1 is the same as the operation in period 1.

1. At the beginning of period 1, you will be paired with the person an experimenter will choose from the other subjects at random. Then, "Input one integer number from 1 to 20" is displayed on the screen of your computer. The following figure is an example of the screen.



Input one integer number from 1 to 20

0～20の整数を入れてください。

2. Please look at the “Payoff table” and confirm the relation between the numbers you and the other person choose and the payoffs.
3. After deciding which number to choose, you input that number in the cell of “The number you chose.” Then, click the “OK” button at the bottom of the screen.
4. After inputting the number you chose on your computer, fill out the number you chose in the column “the number you chose” in the record sheet. Moreover, please fill out why you chose that number in the column “Reason for your decision” in the record sheet.
5. After all subjects click the “OK” button, period 1 is complete. Then, “The number you chose,” “The number the other person chose,” and “Your payoff” are displayed on the screen. We ask that you transcribe this information in the record sheet.
6. After the transcription, please the “NEXT” button at the bottom of the screen.

Once all subjects click the “NEXT” button, period 2 will start. At the beginning of period 2, you will be paired with the person an experimenter will choose from the other subjects at random. The operation after period 1 is the same as the operation in period 1. The experiment is complete when period 20 is complete.

C.4 Rewards

Your rewards are the sum of your payoffs over all 20 periods. For example, if the sum of your payoffs are 2,580, then your rewards are 2,580 yen. If you earn more payoffs in each period, then you receive more rewards.

This concludes our explanation. Next, you can practice making decisions using the “Payoff table for practice.” After practicing, you will have some time to look at the payoff table for the actual experiment before we begin the experiment. If you have any questions, please raise your hand.

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